1. Danae goes cycling on three random mornings each week. When he goes cycling he has eggs for breakfast 70% of the time. When he does not go cycling he has eggs for breakfast 25% of the time.
   a. Create a tree diagram below representing the situation.

   ![Tree Diagram]

   b. Calculate the probability that Danae has eggs for breakfast.

   \[
   \frac{2}{7} \times \frac{7}{10} + \frac{4}{7} \times \frac{1}{4} = \frac{42}{140} + \frac{20}{140} = \frac{62}{140} = \frac{31}{70}
   \]

   Danae will eat eggs, given she goes cycling.

   c. Calculate the probability that it will not rain tomorrow, given it rained today.

   \[
   \frac{P(C \cap E)}{P(C)} = \frac{\frac{21}{70}}{\frac{3}{7}} = \frac{3}{10}
   \]

2. An art gallery has 25 rooms. 16 contain sculptures, 19 contain paintings and only the one contains neither. If a visitor enters a room at random, determine the probability that it contains:
   a. both paintings and sculptures

   ![Venn Diagram]

   \[
   \frac{11}{25}
   \]

   b. only one type of art

   ![Venn Diagram]

   \[
   \frac{13}{25}
   \]

   c. Paintings given that there are no sculptures.

3. The probability that a man will be alive in 25 years is \(\frac{3}{5}\), and the probability that his wife will be alive is \(\frac{2}{3}\). Assuming these events are independent, determine the probability that in 25 years:
   a. both will be alive

   \[
   \frac{3}{5} \times \frac{2}{3} = \frac{2}{5}
   \]

   b. at least one will be alive

   \[
   \frac{2}{5} \times \frac{3}{5} + \frac{2}{5} \times \frac{1}{3} + \frac{2}{5} \times \frac{2}{3} = \frac{2}{15} + \frac{3}{15} + \frac{4}{15} = \frac{13}{15}
   \]

   c. only the wife will be alive.

   \[
   \frac{2}{5} \times \frac{2}{3} = \frac{4}{15}
   \]
4. In a certain town, three newspapers are published. 20% of the population read A, 16% read B, 14% read C, 8% read A and B, 5% read A and C, 4% read B and C, and 2% read all 3 newspapers. A person is selected at random. Use a Venn diagram to help determine the probability that the person reads:

   a. none of the papers \( \frac{65}{280} \) or \( \frac{4}{20} \)
   
   b. at least one of the papers \( \frac{235}{280} \) or \( \frac{7}{20} \)
   
   c. exactly one of the papers \( \frac{68}{280} \) or \( \frac{11}{50} \)
   
   d. either A or B \( \frac{28}{280} \) or \( \frac{7}{20} \)
   
   e. A, given that the person reads at least one paper \( \frac{20}{35} = \frac{20}{5} or \frac{4}{1} \)
   
   f. C, given that the person reads either A or B or both. 

\[ \frac{7}{28} = \frac{1}{4} \]

5. The places were the circus (C), the museum (M) and the park (P). 16 families went to the circus, 22 families went to the museum \( \{ \text{added} \} \), 14 families went to the park, 4 families went to all three places, 7 families went to both the circus and the museum, but not the park, 3 families went to both the circus and the park, but not the museum, 1 family went to the park only.

   a. Draw a Venn diagram to represent the given information using sets labelled C, M and P. Complete the diagram to include the number of families represented in each region.

   b. Find the number of families who
      
      i. went to the circus only \( 2 \)
      
      ii. went to the museum and the park but not the circus; \( 6 \)
      
      iii. did not go to any of the three places on the weekend. \( 12 \)

   c. A family is chosen at random from the group of 40 families. Find the probability that the family went to

      i. the circus; \( \frac{16}{40} = \frac{2}{5} \)
      
      ii. two or more places \( \frac{20}{40} = \frac{1}{2} \)
      
      iii. the park or the circus, but not the museum \( \frac{9}{40} \) or \( \frac{3}{20} \)
      
      iv. the museum, given that they also went to the circus. \( \frac{1}{16} \)

   d. Two families are chosen at random from the group of 40 families. Find the probability that both families went to the circus.

   \[ \frac{16 \times 15}{40 \times 39} = \frac{240}{1560} = \frac{2}{13} \]
6. The weights of corndogs are normally distributed with a mean of 103g and a standard deviation of 13g.

d. Construct a normal distribution graph labeling everything (µ, σ and the name of the graph)! Be precise.

e. Find the percentage of corndogs that weigh between 104g and 129g.

\[
\text{normalcdf}(104, 129, 103, 13) = \frac{44.66}{100} = 44.66\%
\]

f. Calculate the highest weight of the bottom 7% weights of the corndogs. Shade the area in the graph above and label the value.

\[
\text{invnorm}(.07, 103, 13) = 83.81g
\]

g. 700 corndogs are weighed. Calculate the expected number of corndogs that weigh less than 98g.

\[
\text{normalcdf}(-1, 98, 103, 13) = .3563 (700) = 245.18\text{ corndogs}
\]

7. The length of steel rods produced by a machine is normally distributed with a standard deviation of 3 mm. It is found that 2% of all rods are less than 25 mm long. Find the mean length of rods produced by the machine.

\[
\text{invnorm}(0.02, 0.1) = -2.054
\]

\[
\frac{-2.054 \times 3}{3} = \mu = 31.16\text{ mm}
\]

8. Let X be the weight in grams of bags of sugar filled by a machine. Bags less than 500 grams are considered underweight. Suppose that \( X \sim N(503, 2^2) \).

a. What proportion of bags are underweight?

\[
\text{normalcdf}(-1, 500, 503, 2) = .0668
\]

b. Bags weighing more than 507 grams are considered overweight. If the machine fills 6000 bags in one day, how many bags would you expect to be overweight?

\[
\text{normalcdf}(507, 1, 503, 2) = .0275 \times 6000
\]

\[
\approx 136.5 \text{ bags or 137}
\]

9. X is a random variable which is distributed normally with \( \mu = 55 \) and \( \sigma = 7 \). Approximate (no calculator):

a. \( P(48 \leq X \leq 55) \approx 34\%
\]
b. \( P(X \leq 41) \)
\[
= 2.5\% \times 0.025 = 0.0125
\]

c. \( P(X \leq 62) \)
\[
= 68\% \times 32\% \div 2 = 16\% \cdot 0.16
\]
d. \( P(34 \leq X \leq 69) \)
\[
= 0.5 \div 2 = 0.9574
\]

10. Suppose \( X \sim N(30, 8^2) \). Illustrate with a sketch and find \( k \) such that:
   a. \( P(X \leq k) = 0.12 \)
   \[
   \text{invnorm}(0.12, 30, 8) = k \approx 20.60
   \]
   b. \( P(X \geq k) = 0.6 \)
   \[
   P(x \leq k) = 0.4 \]
   \[
   \text{invnorm}(0.4, 30, 8) = 27.97 = k
   \]

11. The distribution curve shown corresponds to age of NBA players and is:
   \( X \sim N(\mu, \sigma^2) \).

Area A = Area B and \( P(A) + P(B) = .18 \)
   a. Find \( \mu \) and \( \sigma \) (hint: z-scores used for \( \sigma \))
   \[
   \mu = 101 + \frac{123}{2} = 112
   \]
   \[
   \text{invnorm}(0.09, 0, 1) = -1.341
   \]
   \[
   -1.341 = \frac{101 - 112}{\sigma}
   \]
   \[
   \sigma = \frac{-11}{1.341} \approx 8.204
   \]
   b. Find age \( k \) for \( P(X > k) = .24 \)
   \[
   P(x < k) = .76
   \]
   \[
   \text{invnorm}(0.76, 112, 8.204) = 117.79
   \]