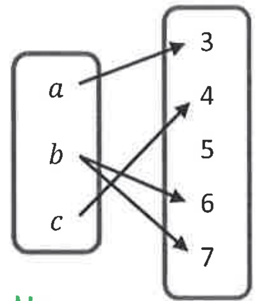
PS6: Functions & Their Graphs (Chapter 1) Review A Name - Key - Date ______ Period ___

Show your work for full credit.

1) Determine if the relation is a function. If not, explain.

a. $\{(2,3),(3,4),(5,1),(6,2),(2,4)\}$
Not a function, $x=2$, $y=3\,\&\,4$

b. $\{(-2,3),(7,3),(1.5,1),(1.1,3),(-52,1)\}$
yes, Its a function

c.


No, ~~b~~ B goes to two values.

2) Determine if the equation represents a function of x. Show your work.

a. $5x+3y^2=17$ $\quad 3y^2=17-5x \quad y=\pm\sqrt{\frac{17-5x}{3}}$, not a function.

b. $\sqrt{x+1}+y=0$ $\quad -\sqrt{x+1}=y$ $\quad$ function!

c. $|y|=4x+1$ (hint: plug in points for x)
$|y|=4(2)+1 \quad |y|=8+1 \quad |y|=9 \quad y=9$ or -9, not a function

3) Evaluation the function as indicated. $f(x)=2x^2-6x$

a. $f(-3) = 18+18 = \boxed{36}$

b. $f(r^3) = \boxed{2r^6-6r^3}$

c. $\frac{f(x+h)-f(x)}{h}$ $\quad f(x+h) = \frac{\cancel{2x^2}+4xh+2h^2-\cancel{6x}-6h-\cancel{2x^2}+\cancel{6x}}{h}$ $-f(x)$ $= \frac{4xh+2h^2-6h}{h} = \boxed{2h+4x-6}$

4) Evaluation the function as indicated. $h(x)=\sqrt{x+3}$.

a. $h(22)$
$h(22)=\sqrt{25}=\boxed{5}$

b. $h(t^2-3)$
$h(t^2-3)=\sqrt{t^2-3+3}=\boxed{t}$

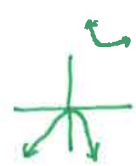
d. $\frac{h(5)}{h(-x)}$ $\quad \frac{\sqrt{8}}{\sqrt{-x+3}} = \frac{\sqrt{8}\sqrt{3-x}}{3-x} = \boxed{\frac{\sqrt{24-8x}}{3-x}}$

5) Find the domain of the functions. Graph the function using a GDC to verify your answer. (don't need to graph by hand) (Use interval notation)

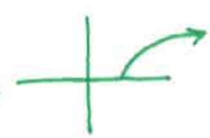
a. $f(x)=x^2+3x-5$
D: $(-\infty,\infty)$

GDC graph.

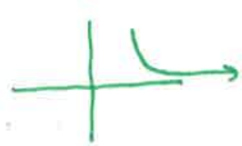
b. $f(x)=\frac{4x^2}{3x-6}$
$3x-6\neq 0$
$3x\neq 6$
$x\neq 2$
D: $(-\infty,2)\cup(2,\infty)$



c. $f(x)=\sqrt{2x-4}$
$2x-4\geq 0$
$x\geq 2$
D: $[2,\infty)$

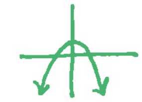


d. $f(x)=\frac{3}{\underbrace{\sqrt{x-5}}_{\neq 0}}$ $\rightarrow$ so
$\sqrt{x-5}$
$x-5\geq 0$
$x-5>0$
$x>5$
D: $(5,\infty)$

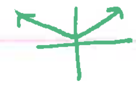


6) Find the domain and range of the functions. Use a graph to help you find the range.

a. $f(x) = 1 - 3x^2$ D: $\mathbb{R}$ or $(-\infty, \infty)$ R: [redacted] $\boxed{y \le 1}$



b. $f(x) = \sqrt{x^2 + 3}$ D: $(-\infty, \infty)$ R: $[\sqrt{3}, \infty)$ or $[\sim 1.73, \infty)$ $\boxed{y \ge \sqrt{3}}$



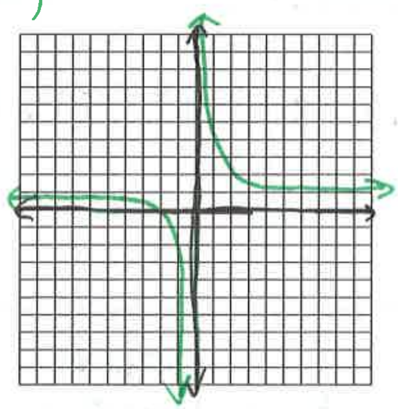
c. $f(x) = |2x + 3| - 9$ D: $(-\infty, \infty)$ R: $[-9, \infty)$ $\boxed{y \ge -9}$

7) Graph the equation using a GDC and use the vertical line test to decide if it is a function. State the domain and range if it is a function. Make a quick sketch of the graph (no need to plot points).

a. $y = \frac{x+3}{x}$ ✓ Function

D: $(-\infty, 0) \cup (0, \infty)$

R: $y > 1$ or $y < 1$



c. $3x + y^3 = 8$

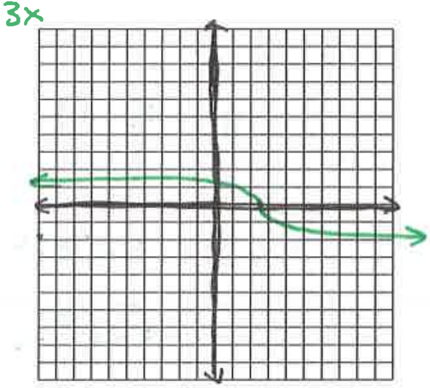
$y^3 = -3x + 8$

$y = \sqrt[3]{8 - 3x}$

Function ✓

D: $\mathbb{R}$

R: $\mathbb{R}$

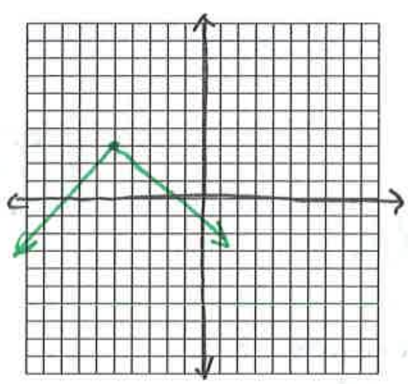


b. $y = 3 - |x + 5|$

D: $\mathbb{R}$

R: $y \le 3$

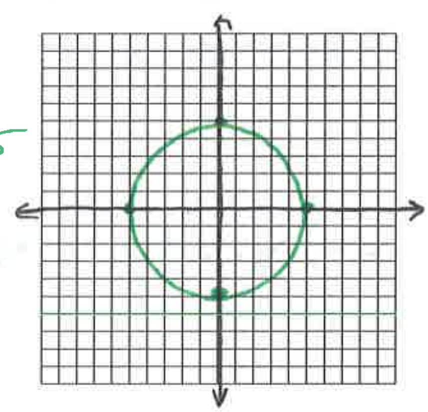
Function ✓



e. $x^2 + y^2 = 25$ Not a function.

D: $[-5, 5]$

R: $-5 \le y \le 5$



8) Determine a) what interval the function is increasing, decreasing or constant, b) if the function is odd, even or neither c) and if there are any local maximum or minimum values. Make a quick sketch of the graph.

a. $h(x) = \frac{x}{x^2+1}$

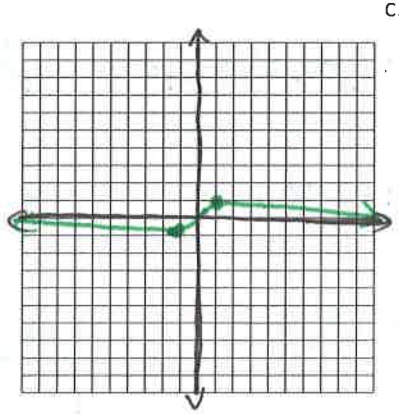
inc: $(-1, 1)$

Dec: $(-\infty, -1) \cup (1, \infty)$

min: $(-1, -\frac{1}{2})$

max: $(1, \frac{1}{2})$

odd function



c. $f(x) = x^3 - x$

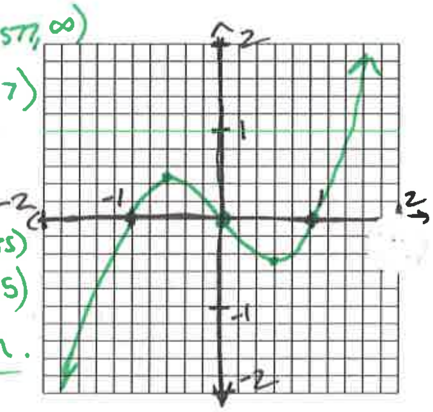
inc: $(-\infty, -.577) \cup (.577, \infty)$

Dec: $(-.577, .577)$

min: $(.577, -.385)$

max: $(-.577, .385)$

odd function.



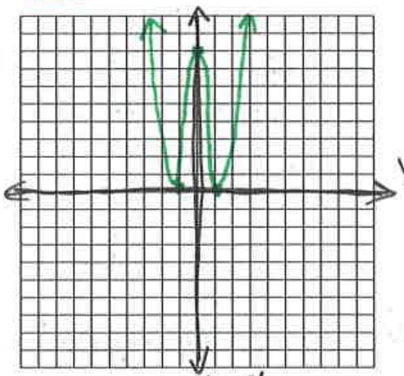
9) Determine a) what interval the function is increasing, decreasing or constant, b) if the function is odd, even or neither c) and if there are any local maximum or minimum values. Make a quick sketch of the graph.

a. $f(x) = (x^2 - 4)^2$

inc: $(-2,0) \cup (2,\infty)$ max: $(0,16)$

dec: $(-\infty,-2) \cup (0,2)$ mins $(-2,0)$ & $(2,0)$

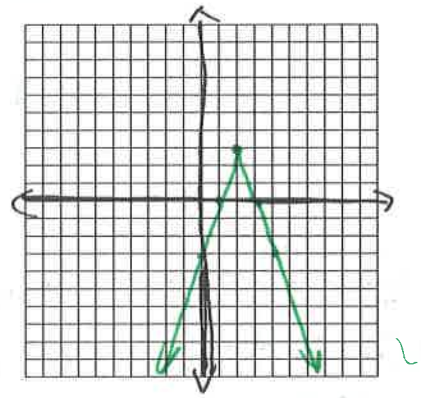
Even Function



c. $f(x) = -3|x-2| + 3$

inc: $(-\infty,2)$ max: $(2,3)$

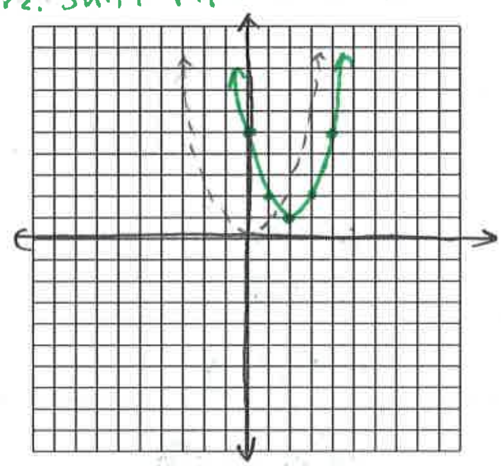
Dec: $(2,\infty)$ Neither odd or even.



10) Sketch the graph by hand and name the transformations and the base graph.

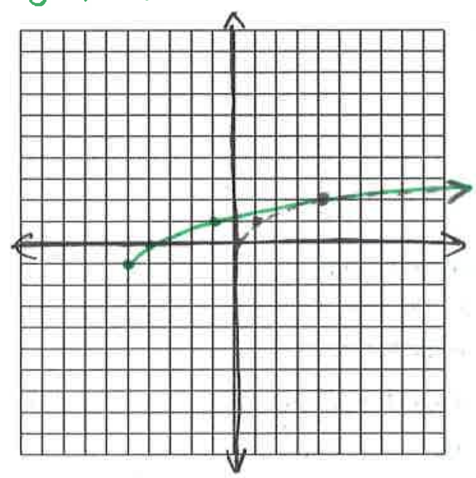
a. $f(x) = (x-2)^2 + 1$

vert shift up 1 unit
Horz. shift right 2 units.



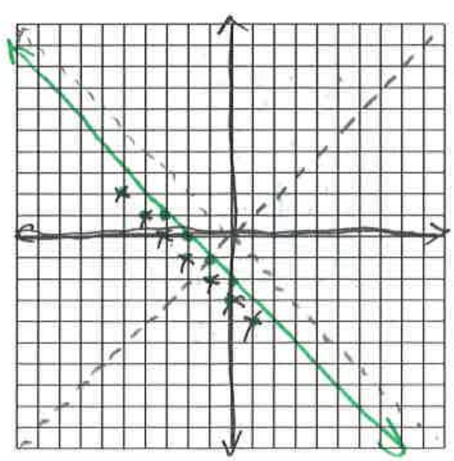
b. $f(x) = \sqrt{x+5} - 1$

Down 1 unit
Left 5 units.



c. $f(x) = -(x+3) + 1$

reflection over x
Left 3 units,
up 1 unit.

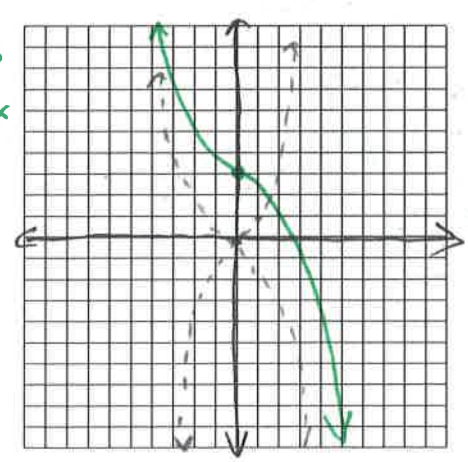


e. $f(x) = \frac{1}{-2}(x^3 - 4) + 1$

$= -\frac{1}{2}x^3 + 2 + 1$

$= -\frac{1}{2}x^3 + 3$

reflection over x
1/2 shrink
up 3 units



11) Let $f(x) = 2x^2 + 2$, $g(x) = \sqrt{x-2}$ and $h(x) = 2x^2 - 2$. Find and state the domain of:

a. $(f - h)(x)$

$(f-h)(x) = f(x) - h(x)$

$(f-h)(x) = 2x^2 + 2 - 2x^2 + 2$

$= \boxed{4}$

$\boxed{D: \mathbb{R}}$

b. $(fg)(x)$

$fg(x) = f(x)\, g(x)$

$\boxed{fg(x) = (2x^2+2)(\sqrt{x-2})}$

Domain $x - 2 \geq 0$

$x \geq 2$

$\boxed{D: [2, \infty)}$

c. $\left(\frac{h}{g}\right)(x)$

$\frac{h(x)}{g(x)} = \frac{2x^2-2}{\sqrt{x-2}} \cdot \frac{\sqrt{x-2}}{\sqrt{x-2}}$

$\boxed{= \frac{(2x^2-2)\sqrt{x-2}}{x-2}}$

Domain: $x - 2 \neq 0$, $x \neq 2$

$x - 2 \geq 0$, $x \geq 2$, so...

$\boxed{D: (2, \infty)}$

d. $(f \circ g)(x)$

$f(g(x)) = 2\left(\sqrt{x-2}\right)^2 + 2$

$= 2(x-2) + 2$

$= 2x - 4 + 2$

$\boxed{= 2x - 2}$

$\boxed{D: [2, \infty)}$

e. $g(h(x)) = \sqrt{2x^2 - 2 - 2}$

$\boxed{= \sqrt{2x^2 - 4}}$

D: $2x^2 - 4 \geq 0$

$x^2 \geq 2$

$\pm\sqrt{\ }$ neg flips the sign.

$x \geq \sqrt{2}$ or $x \leq -\sqrt{2}$

D: $(-\infty, -\sqrt{2}] \cup [\sqrt{2}, \infty)$

f. find $g(f(3))$.

$g(f(x)) =$

$g(f(x)) = \sqrt{(2x^2+2) - 2}$

$g(f(x)) = \sqrt{2} \cdot x$

$\boxed{g(f(3)) = 3\sqrt{2}}$

$\boxed{D: \mathbb{R}}$

12) Find the inverse of f **algebraically** if it exists. If it doesn't exist, Show the graph fails the horizontal line test. If it does, Sketch f *and* f^{-1} as well as the **identity function**.

a. $f(x) = \frac{x-6}{x+2}$

$(y+2)\, x = \frac{y-6}{y+2}(y+2)$

$yx + 2x = y - 6$

$-yx + 6 \quad -yx + 6$

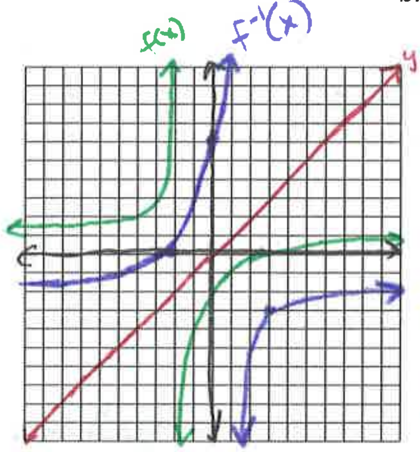
$6 + 2x = y - yx$

$\frac{6+2x}{1-x} = \frac{y(1-x)}{1-x}$

$y = \frac{6+2x}{1-x}$

$\boxed{f^{-1}(x) = \frac{6+2x}{1-x}}$

Function!



b. $f(x) = \sqrt{2x-2}$ $\quad y \geq 0$, $x \geq 1$

$x = \sqrt{2y-2}$

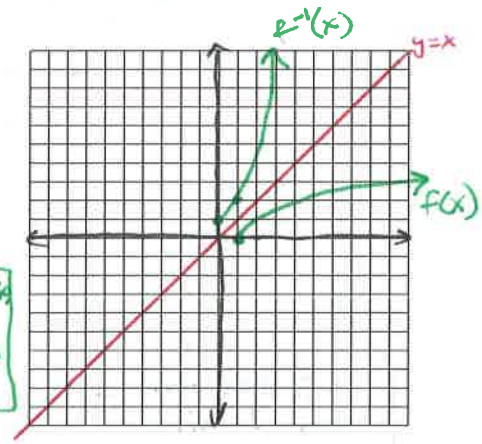
$x^2 = 2y - 2$

$\frac{x^2+2}{2} = \frac{2y}{2}$

$\frac{1}{2}x^2 + 1 = y = f^{-1}(x)$

$y \geq 1$, $x \geq 0$

Function!



c. $f(x) = -2x^2 + 3$

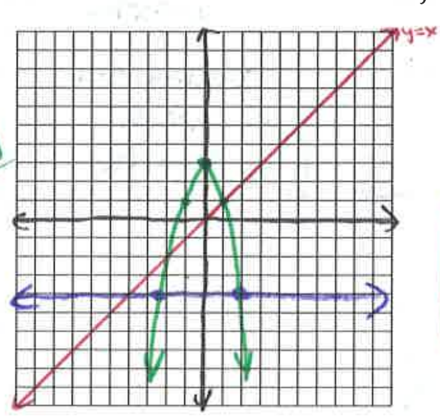
$x = -2y^2 + 3$

$\frac{x-3}{-2} = y^2$

$\pm\sqrt{\frac{x-3}{-2}} = y$

Not a function.

Fails H.L.T.



d. $f(x) = \sqrt[3]{x-4}$

$x = \sqrt[3]{y-4}$

$x^3 = y - 4$

$x^3 + 4 = y$

$\boxed{x^3 + 4 = f^{-1}(x)}$

Function!