1.1 LINES (NOT GOING OVER)
1.2 FUNCTIONS
1.3 GRAPHS

I CAN LEARN HOW TO EVALUATE FUNCTIONS AND FIND THEIR DOMAINS.

VOCABULARY

- **FUNCTION** A FUNCTION *F* FROM SET A TO SET *B* IS A RELATION THAT ASSIGNS TO EACH ELEMENT *X* IN THE SET *A* EXACTLY ONE ELEMENT *Y* IN THE SET *B*.
- **DOMAIN** THE SET OF INPUTS OF THE FUNCTION f.
- **RANGE** THE SET OF ALL OUTPUTS FOR THE GIVEN SET OF INPUTS OF THE FUNCTION f.
- **INDEPENDENT VARIABLE** A VARIABLE IN AN EQUATION THAT REPRESENTS A FUNCTION THAT CAN TAKE ON ANY VALUE FOR WHICH THE FUNCTION IS DEFINED.
- **DEPENDENT VARIABLE** A VARIABLE IN AN EQUATION THAT REPRESENTS A FUNCTION WHOSE VALUE DEPENDS ON THE VALUE OF THE INDEPENDENT VARIABLE.
- IN THIS CASE, THE VARIABLE REPRESENTING THE DOMAIN ELEMENTS (*X*) IS CALLED THE INDEPENDENT VARIABLE. THE VARIABLE REPRESENTING THE RANGE ELEMENTS (*Y*) IS CALLED THE DEPENDENT VARIABLE.

FUNCTIONS

- A RULE OF CORRESPONDENCE THAT PAIRS ITEMS FROM ONE SET WITH ITEMS FROM A DIFFERENT SET IS A RELATION .
- IN FUNCTIONS THAT CAN BE REPRESENTED BY ORDERED PAIRS, THE FIRST COORDINATE IN EACH ORDERED PAIR IS THE INPUT AND THE SECOND COORDINATE IS THE OUTPUT.
  - **DOMAIN:** THE SET OF X-VALUES
  - **RANGE:** THE SET OF Y-VALUES.
  - SOME CHARACTERISTICS OF FUNCTIONS ARE . . .
- 1) EACH ELEMENT IN THE DOMAIN MUST BE MATCHED WITH AN ELEMENT OF THE RANGE.
- 2) SOME ELEMENTS IN THE RANGE MAY NOT BE MATCHED WITH ANY ELEMENT IN THE DOMAIN.
- 3) TWO OR MORE ELEMENTS OF THE DOMAIN MAY BE MATCHED WITH THE SAME ELEMENT OF THE RANGE.

IS IT A FUNCTION?

- TO DECIDE WHETHER A RELATION IS A FUNCTION, DECIDE WHETHER EACH INPUT VALUE IS MATCHED WITH EXACTLY ONE OUTPUT VALUE.
- IF ANY INPUT VALUE OF A RELATION IS MATCHED WITH TWO OR MORE OUTPUT VALUES, THE RELATION IS NOT A FUNCTION.
- SOME COMMON WAYS TO REPRESENT FUNCTIONS ARE . . .

  1) VERBALLY IN A SENTENCE

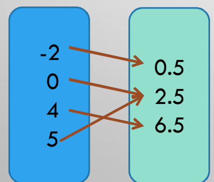
  2) NUMERICALLY IN A TABLE OR LIST OF ORDERED PAIRS

  3) GRAPHICALLY BY POINTS ON A GRAPH IN THE COORDINATE PLANE

  4) ALGEBRAICALLY BY AN EQUATION IN TWO VARIABLES

PROBLEM 1:

- IDENTIFY THE DOMAIN AND RANGE OF EACH RELATION. REPRESENT THE RELATION WITH A MAPPING DIAGRAM. IS IT A FUNCTION?
- {(-2,0.5), (0,2.5), (4,6.5), (5,2.5)}
- MAPPING DIAGRAM:

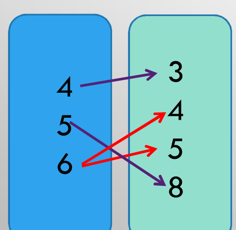


If each domain value is mapped to only one range value then the relation is a function.

It's A Function!

PROBLEM 2:

- IDENTIFY THE DOMAIN AND RANGE OF EACH RELATION. REPRESENT THE RELATION WITH A MAPPING DIAGRAM. IS IT A FUNCTION?
- {(6,5), (4,3), (6,4), (5,8)}
- MAPPING DIAGRAM:



If each domain value is mapped to only one range value then the relation is a function.

Not A Function

IS IT A FUNCTION?

- **EXAMPLE 3:** DECIDE WHETHER THE TABLE REPRESENTS *Y* AS A FUNCTION.

x	**-3**	**-1**	**0**	**2**	**4**
y	5	-12	5	3	14

- DOES EACH VALUE OF x GO TO ONLY ONE VALUE OF y?

Function!

D: {-3, -1, 0, 2, 4}

R: {-12, 3, 5, 14}

EXAMPLE 4. WHICH OF THE FOLLOWING EQUATIONS REPRESENTS *Y* AS A FUNCTION OF *X*?

- $2x^2 - y + 1 = 0$

+y +y

$y = 2x^2 + 1$

Function!

- $x + y^2 - 6 = 0$

+6 +6

$x + y^2 = 6$

−x −x

$\pm\sqrt{y^2} = \pm\sqrt{6-x}$

$y = \pm\sqrt{6-x}$

Not a function!

Ex: $y = \pm\sqrt{6-0}$

$y = \pm\sqrt{6}$

$y = \sqrt{6}$

$y = -\sqrt{6}$

FUNCTION NOTATION

- THE SYMBOL $f(x)$ IS **FUNCTION NOTATION** FOR THE VALUE OF f AT x OR f OF x, USED TO DESCRIBE y AS A FUNCTION OF x. IN THIS CASE, *F* IS THE NAME OF THE FUNCTION AND $f(x)$ IS THE OUTPUT VALUE OF THE FUNCTION AT THE INPUT VALUE x.
- LETS RECAP: WHAT ARE THE DIFFERENT TERMS FOR Y AND X?
- $\boldsymbol{y-value(s)}$: Range (list), dependent, output, $f(x)$, $g(t)$, $N(w)$
- $\boldsymbol{x-value(s)}$: Domain, Independent, input, x, t, v, w, r, n

$(x+h)^2 \neq x^2+h^2$

EXAMPLE 5.

- $f(x) = x^2 - 4x$, FIND THE FOLLOWING. (HINT: C NEEDS TO BE FOILED)

A) $f(3) = 3^2 - 4(3)$

$= 9 - 12$

$f(3) = -3$

B) $f(a) = a^2 - 4a$

C) $f(x+h) = (x+h)^2 - 4(x+h)$

$f(x+h) = x^2 + 2xh + h^2 - 4x - 4h$

hint: S.C

$(a+b)^2 = (a+b)(a+b)$

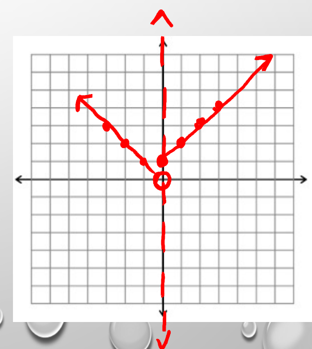
$a^2 + 2ab + b^2$

WHAT DID YOU GET?

EXAMPLE 6

Graphically it changes

- $f(x) = \begin{cases} x+1, if\ x \geq 0 \\ -x\ if\ x < 0 \end{cases}$, FIND THE FOLLOWING:
- A) $f(2) =$ 3
- B) $f(-5) =$ 5



INTERVAL NOTATION

- **INTERVAL NOTATION** IS A NOTATION USED TO SHOW A SET OF NUMBERS.
- WE USE PARENTHESES TO NOT INCLUDE AN ENDPOINT. WE USE BRACKETS TO INCLUDE AN ENDPOINT. WE USE INTERVAL NOTATION TO DESCRIBE DOMAIN.
- EXAMPLE: $(3,9)$ MEANS ALL VALUES BETWEEN 3 AND 9, NOT INCLUDING 3 OR 9.
- AS AN INEQUALITY THIS WOULD BE $3 < x < 9$
- EXAMPLE: $[-2,9)$ MEANS ALL VALUES BETWEEN -2 AND 9, INCLUDING -2.
- THE EQUIVALENT INEQUALITY IS $-2 \leq x < 9$.
- WHAT ABOUT $x < 4$. HOW IS THAT REPRESENTED IN INTERVAL NOTATION?
- WE HAVE TO DEAL WITH INFINITY. WE USE + OR − FOR DIRECTION AND WE **NEVER INCLUDE INFINITY.** $(-\infty, 4)$
- WHAT ABOUT $-9 \leq x$? $[-9, \infty)$

DOMAIN OF A FUNCTION

- THE **IMPLIED DOMAIN** OF A FUNCTION IS THE SET OF ALL X SUCH THAT THEIR CORRESPONDING *Y* IS A REAL NUMBER. AT LEAST FOR A WHILE, WE WILL ONLY CONSIDER THREE SITUATIONS,
- 1. POLYNOMIALS ⇒ DOMAIN IS $(-\infty, \infty)$ (AKA $\mathbb{R}$)
- 2. FRACTIONS ⇒ CANNOT HAVE ANY NUMBER IN THE DOMAIN THAT MAKES THE DENOMINATOR ZERO.
- 3. RADICALS ⇒ IF THE INDEX IS EVEN (EX. $\sqrt[4]{x}$) THEN THE RADICAND MUST BE NONNEGATIVE. WE SET WHAT'S UNDER THE RADICAL ≥ 0 AND SOLVE.
- **RANGE: THESE ARE ALL THE POSSIBLE Y VALUES (OUTCOMES):** THIS CAN BE FOUND GRAPHICALLY OR BY PLUGGING X VALUES IN. SOMETIME THERE ARE TRICKS TO FINDING THE RANGE. WE WILL VISIT THOSE LATER, BUT THIS…
- TRICK #1: WE CAN THINK; WHAT OUTCOMES WILL NEVER HAPPEN?

EXAMPLES

a^2-b^2

$(a-b)(a+b)$

- FIND THE DOMAIN OF THE FOLLOWING FUNCTIONS.

① $f(x) = x^3 + 3x + 1$

D. $\mathbb{R}$, $(-\infty,\infty)$

② $f(x) = \frac{2}{x+2}$

D: $\mathbb{R}, x \neq -2$ or $(-\infty,-2)\cup(-2,\infty)$

- $f(x) = \sqrt{5-x}$

$5-x \geq 0$

$-5 \quad -5$

$-x \geq -5$

$\div -1 \quad \div -1$

$x \leq 5$

D: $(-\infty,5]$

- $f(x) = \sqrt{x^2-1}$

$x^2-1 \geq 0$

$+1 \quad +1$

$x^2 \geq 1$

$x \geq 1 \qquad x \leq -1$

$(-\infty,-1]\cup[1,\infty)$

THE DIFFERENCE QUOTIENT

- THE DIFFERENCE QUOTIENT IS A RATIO REPRESENTED BY $\frac{f(x+h)-f(x)}{h}, h \neq 0.$
- THIS FORMULA COMPUTES THE SLOPE OF THE SECANT LINE THROUGH TWO POINTS ON THE GRAPH OF f. THESE ARE THE POINTS WITH X-COORDINATES x AND $x + h$. THE DIFFERENCE QUOTIENT IS USED IN THE DEFINITION OF THE DERIVATIVE.
- LETS USE IT!

EXAMPLE

- FOR $f(x) = -2x + 4$ FIND $\frac{f(x+h)-f(x)}{h}$

$f(x+h) = -2(x+h)+4$

$$\frac{-2(x+h)+4-(-2x+4)}{h}$$

$$\frac{-2x-2h+4+2x-4}{h}$$

$$\frac{-2h}{h} = -2$$

ANOTHER EXAMPLE

- FOR $f(x) = x^2 - 4x + 7$ FIND $\frac{f(x+h) - f(x)}{h}$

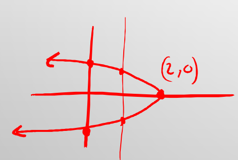
HOMEWORK FOR THIS SECTION

- P.24 #8-16EVEN, 19-47ODD, 55-65ODD, (73, 75 IS A CHALLENGE PROBLEM), 84

(12)

x	0	1	2	1	0
y	-4	-2	0	2	4

Not a function
Ex: Input 1 has two outputs.
x=0 & y=-4,4



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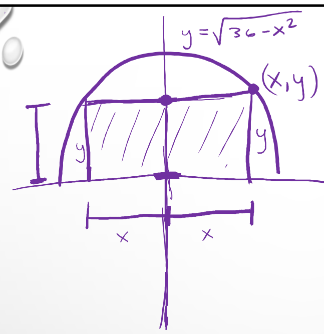


$A = l \cdot w$

$l = y$, $w = 2x$

$A = 2xy$

$A = 2x\sqrt{36-x^2}$

Check Domain

$A = 2(-4)\sqrt{36-(-4)^2}$

$\sqrt{36-16}$

$A = -8\sqrt{20}$

$\approx -8 \cdot 4.5$

$A \approx -36$

$[-6, 6]$

$-6 \le x \le 6$

~~-4~~

$0 < x \le 6$

$36 - x^2 \ge 0$

$\frac{-x^2}{-1} \ge \frac{-36}{-1}$

$\pm\sqrt{x^2} \le \sqrt{36}$

$x \le 6 \quad x \ge -6$

D: $0 < x < 6$

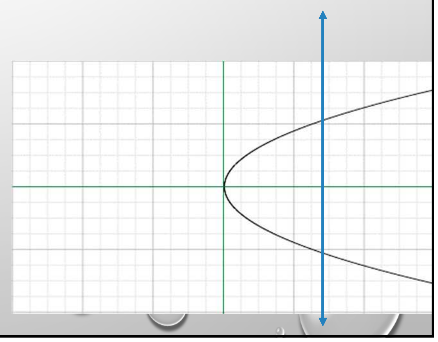
$(0, 6)$

1.3 GRAPH OF A FUNCTION

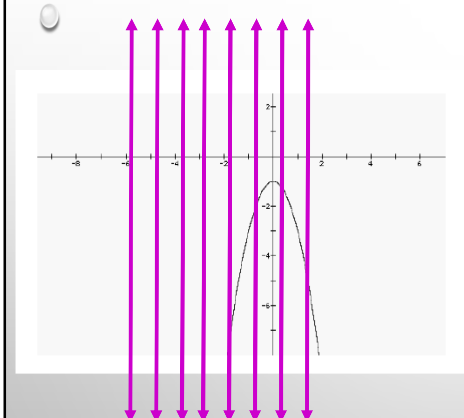
SECTION OBJECTIVES: STUDENTS WILL KNOW HOW TO ANALYZE GRAPHS OF FUNCTIONS.

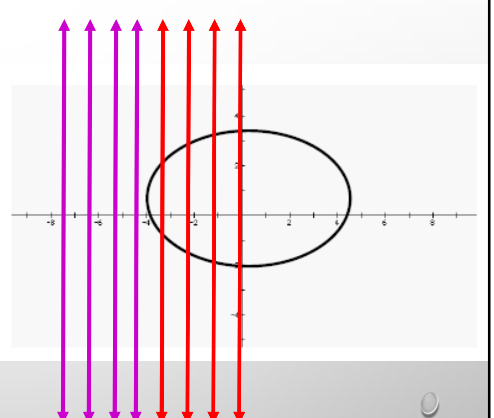
1.3 LETS GO! GRAPH OF A FUNCTION

- HOW CAN WE TELL THE GRAPH IS A FUNCTION? SINCE $y = f(x)$, GRAPHING A FUNCTION IS NO DIFFERENT THAN GRAPHING EQUATIONS IN TWO VARIABLES.
- DRAW THE GRAPH $y^2 = x$ YOU MAY NEED TO USE A T CHART.
- TO TELL IF THE GRAPH IS A FUNCTION YOU NEED TO USE THE <u>VERTICAL LINE TEST</u>
- IF EVERY VERTICAL LINE MEETS THE GRAPH OF A RELATION IN AT MOST ONE POINT, THEN THE RELATION IS A FUNCTION.

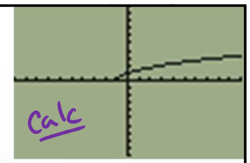


IS THE GRAPH A FUNCTION?





DOMAIN AND RANGE



$20 = \sqrt{x+1}$
$400 = x+1$
$399 = x$

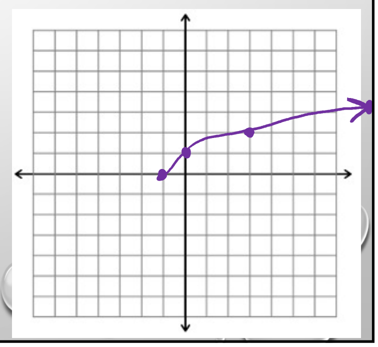
- WHAT IS THE DOMAIN OF A FUNCTION? ALL POSSIBLE X VALUES! USE INTERVAL NOTATION FROM NOW ON.
- EX: WHAT IS THE DOMAIN AND RANGE OF $f(x) = \sqrt{x+1}$

$x + 1 \geq 0$
$x \geq -1$
$[-1, \infty)$

- WE CAN DO THIS ALGEBRAICALLY AND GRAPHICALLY. GRAPHICALLY, RANGE IS FROM THE $y - axis$ AND DOMAIN FROM THE $x - axis.$
- DOMAIN: $[-1, \infty)$
- RANGE: $y \geq 0$

R: $[0, \infty)$

X	$f(x)$
-2	ERR
-1	0
3	2
8	3



INCREASING AND DECREASING FUNCTIONS BY EYE.

- WE ALWAYS LOOK FROM LEFT TO RIGHT TO IDENTIFY WHETHER A FUNCTION IS INCREASE OR DECREASING.
- EXAMPLE: $f(x) = x^3 + 3x^2$

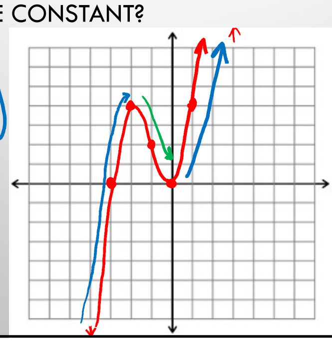
$-64 + 3(16)$ | $-27 + 27$ | $27+27$
$-64 + 48$ | $-8 + 12$
-16 | $-1 + 3$ | $1+3$

- WHEN LOOKING AT THIS GRAPH, WE WANT TO ONLY LOOK AT WHAT THE Y VALUE IS DOING AS THE X VALUES GO FROM LEFT TO RIGHT.
- WHAT INTERVAL IS THE FUNCTION INCREASING OR DECREASING? CAN IT BE CONSTANT?

X-values

INC: $(-\infty, -2) \cup (0, \infty)$

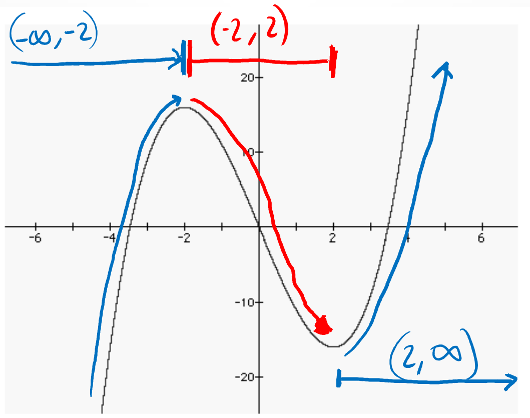
DEC: $(-2, 0)$



X	Y
-4	-16
-3	0
-2	4
-1	2
0	0
1	4
3	54

ANSWER

- **EXAMPLE 3.** DETERMINE THE OPEN INTERVALS ON WHICH $f(x) = x^3 - 12x$ IS
- INCREASING OR DECREASING.



f is increasing on $(-\infty, -2) \cup (2, \infty)$

f is decreasing on $(-2, 2)$

DEFINITION OF INCREASING, DECREASING AND CONSTANT FUNCTIONS

THE FORMAL DEFINITION IS ONE USING POINTS. MY DEFINITIONS FOLLOW.

1. A FUNCTION *F* IS **INCREASING** ON AN INTERVAL IF, FOR ANY x_1 AND x_2 IN THE INTERVAL SUCH THAT $x_1 < x_2$, THEN $f(x_1) < f(x_2)$.

SAPUTO SAYS: AS X GETS BIGGER Y GETS BIGGER, THIS IS INCREASING.

2. A FUNCTION *F* IS **DECREASING** ON AN INTERVAL IF, FOR ANY x_1 AND x_2 IN THE INTERVAL SUCH THAT $x_1 < x_2$, THEN $f(x_1) > f(x_2)$.

SAPUTO SAYS: AS X GETS BIGGER Y GETS SMALLER, THIS IS DECREASING.

3. A FUNCTION *F* IS **CONSTANT** ON AN INTERVAL IF, FOR ANY x_1 AND x_2 IN THE INTERVAL, THEN $f(x_1) = f(x_2)$.

SAPUTO SAYS: AS X GETS BIGGER OR SMALLER, Y STAYS THE SAME, THIS IS CONSTANT.

RELATIVE MINIMUM AND MAXIMUM VALUES

- DEFINITION: F(A) IS A RELATIVE MAXIMUM IF $f(x_1) < f(a)$ AND WHEN $f(x_2) < f(a)$ WHERE $x_1 < a$ AND $x_2 > a$.
- F(A) IS A RELATIVE MINIMUM IF $f(x_1) > f(a)$ AND WHEN $f(x_2) > f(a)$ WHERE $x_1 < a$ AND $x_2 > a$.
- WE CAN DO THIS BY USING THE MAX AND MIN BUTTON ON THE CALCULATOR OR BY GRAPHING AND OBSERVING.

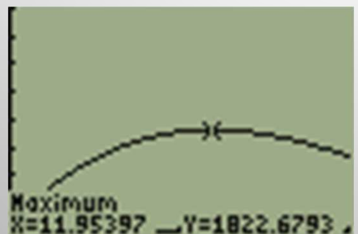
EXAMPLE 4.

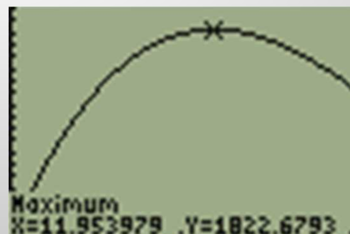
- GRAPH $f(x) = -3x^2 - 2x + 1$ ON YOUR GRAPHING CALCULATOR. USE THE MAX AND MIN FEATURE TO FIND THE RELATIVE MAX AND MIN.

EXAMPLE 5

- **EXAMPLE 5.** THE PROFIT FOR A NEW COMPANY CAN BE MODELED BY
- $P = 0.225x^3 - 17.21x^2 + 315x + 132.1$

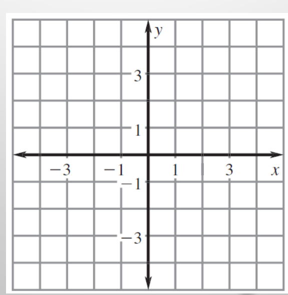
WHERE *P* IS IN THOUSANDS OF DOLLARS AND *X* IS THE NUMBER OF UNITS SOLD IN THOUSANDS. WHAT WOULD BE THE MAXIMUM PROFIT FOR THIS COMPANY?

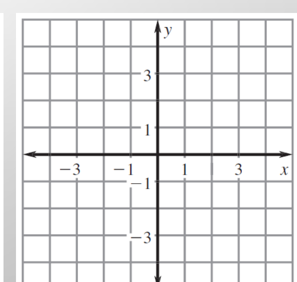




GRAPHING STEP FUNCTIONS AND PIECEWISE-DEFINED FUNCTIONS

- GREATEST INTEGER FUNCTION: GREATEST INTEGER FUNCTION, DENOTED BY [|X|], IS
- [|X|] = THE GREATEST INTEGER LESS THAN OR EQUAL TO X. *THIS IS ALSO CALLED A STEP FUNCTION.*
- *PIECE WISE FUNCTIONS. TWO RELATIONS THAT HAVE RESTRICTIONS ON X TO FORM ONE FUNCTION.*
- $f(x) = \begin{cases} x + 1, if\ x \leq 2 \\ 1 - x, if\ x > 2 \end{cases}$





EVEN AND ODD FUNCTIONS

- 1. A FUNCTION *F* IS EVEN IF, FOR EACH *X* IN THE DOMAIN OF *F*, $f(-x) = f(x)$.
- 2. A FUNCTION *F* IS ODD IF, FOR EACH *X* IN THE DOMAIN OF *F*, $f(-x) = -f(x)$.

SAPUTO SAYS: HERE IS THE TRICK!!! PLUG IN –X AND SEE WHAT HAPPENS WHEN YOU EVALUATE IT!

TEST THE FOLLOWING FUNCTIONS FOR BEING ODD OR EVEN.

$$f(x) = \frac{x}{x^2 + 1}$$

YOU TRY: IF THE FOLLOWING FUNCTION ODD OR EVEN.

- $f\,(x) = x^4 - |x|$
- $f\,(x) = x^3 + 2x^2$

EVEN AND ODD GRAPHICALLY

- EVEN FUNCTION ARE SYMMETRIC TO THE Y AXIS. $y = x^2$ IS EVEN BECAUSE IT HAS SYMMETRY OVER THE Y-AXIS.
- ODD FUNCTION HAVE SYMMETRY OVER BOTH Y AND X, WHICH IS LIKE SAYING THEY HAVE SYMMETRY OVER THE ORIGIN. GRAPH $y = x^3 - 3x$. NOTICE THAT EVERY POINT CAN BE REFLECTED OVER THE ORIGIN AND LIE ON THE LINE.
- WARNING: BE CAREFUL WITH ODD EXPONENT FUNCTIONS. THEY MY LOOK ODD, BUT REALLY THEY ARE NEITHER.

HOMEWORK FOR THIS SECTION

- P. 28 #83,84 P. 37#7-75EVERY OTHER ODD(NO 47 OR 51), 87,99-104