Chapter 1.4 Transformations but first 1.3 Recap

Section Objectives: Students will know how to analyze graphs of functions.

PS6.4

Recap of Important information

<u>1.2 Functions and their Graphs</u>

- Vertical line test for functionality
- Domain (x values) and range (y values)
- Increasing, constant and decreasing functions. Identifying the interval
- Relative max and min. Use of a calculator.
- **Step functions & piecewise functions**
- **Even and odd function**

Graphing Piecewise functions: Recap

- Graph the following and state the domain and range

⑤ D: $\mathbb{R}$

R: $(\infty,4) \cup [3] \cup$ $[5,-\infty)$

- $f(x) = \begin{cases} x^2, & when\ x < -2 \\ 3, & when\ -2 \le x < 2 \\ -2x-1, & when\ x \ge 2 \end{cases}$

hollow

① Inflections

② graph x^2 @ -2 and below

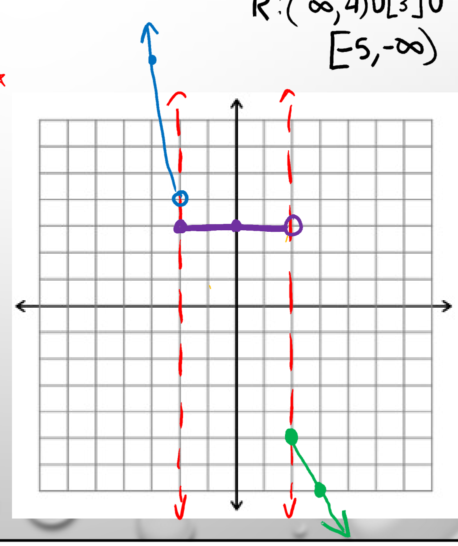
x	y
-2	4
-3	9
-4	16

③ graph 3 between -2 & 2

x	y
-2	3
0	3
2	3

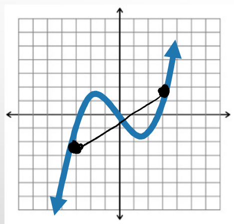
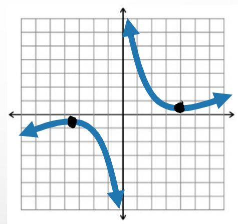
④ -2x-1 @ 2 & greater

x	y
2	-5
3	-7
4	-9



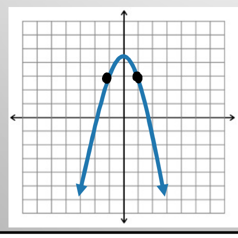
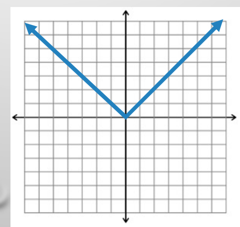
Odd and Even Function: Recap

- We can see graphically and algebraically whether a function is odd or even.
- **Graphically odd:** A function is odd if there is symmetry over the origin.
- **Examples:**

$f(-x) = -f(x)$

- **Graphically Even:** A function is even if it has symmetry over the y-axis.
- **Examples:**

$f(-x) = f(x)$

1.4 Shifting, Reflecting and Stretching

Section Objectives: Students will recognize graphs of common functions, use vertical and horizontal shifts and reflections to sketch graphs of functions, and use nonrigid transformations to sketch a graph.

1.3 Summary of Graphs of Common Functions

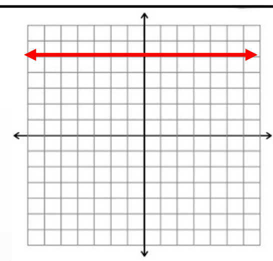
There are many functions that exist. We need to be familiar with the commons ones that show up most often.

Be familiar with the following functions:

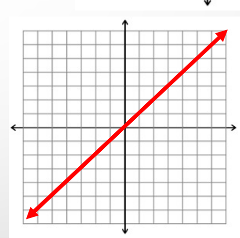
- $f(x) = c$ Constant function (Not really going to be used much.)
- $\boldsymbol{f(x) = x}$ **Identity (linear) function**
- $\boldsymbol{f(x) = |x|}$ **Absolute value function**
- $\boldsymbol{f(x) = x^2}$ **Quadratic function**
- $\boldsymbol{f(x) = x^3}$ **Cubic function**
- $\boldsymbol{f(x) = \sqrt{x}}$ **Square root function**
- $\boldsymbol{f(x) = \frac{1}{x}}$ **Rational functions**

Lets break these down individually and look at their shapes.

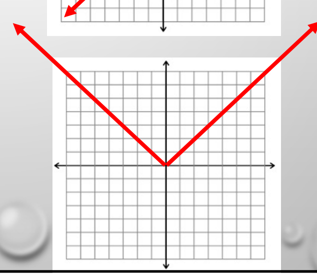
- $f(x) = c$ Constant function (ex: $f(x) = 5$)



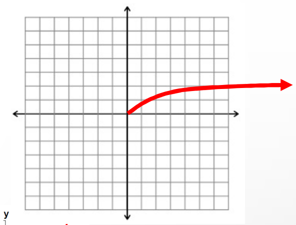
- $f(x) = x$ Identity function



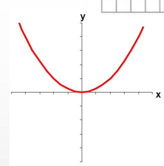
- $f(x) = |x|$ Absolute value function



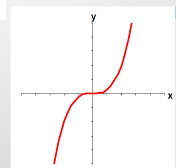
- $f(x) = \sqrt{x}$ Square root function



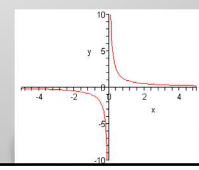
- $f(x) = x^2$ Quadratic function



- $f(x) = x^3$ Cubic function

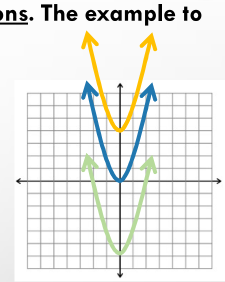


- $f(x) = \frac{1}{x}$ Rational function.



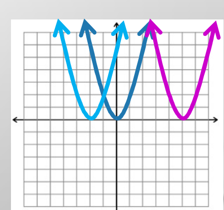
Vertical and Horizontal Shifts

Vertical shift moves one of the common graphs up or down when you add or subtract a number to it (called a constant). These are called <u>rigid transformations</u>. The example to the right is of the common function x^2.



- $\underline{h(x) = f(x) + c}$ This is a vertical shift up c units. The yellow represents a shift 4 units up from the common function. $f(x) = x^2 + 4$
- $\underline{h(x) = f(x) - c}$ This is a vertical shift down c units. The green represents a shift down 6 units from the common function. $f(x) = x^2 - 6$.

Horizontal Shifts moves the graph left or right. This happens when you add or subtract a number from the x value. The shifts work oppositely. Example is for $f(x) = x^2$

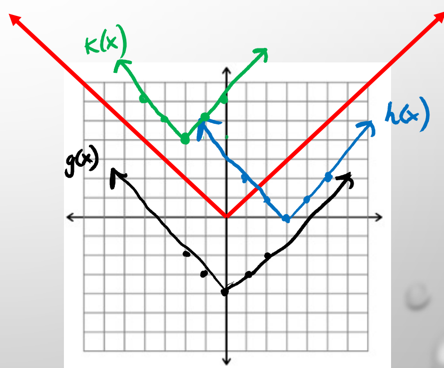


- $\underline{\boldsymbol{h(x) = f(x - c)}}$ This is a horizontal shift **right** c units. The purple represents a shift 5 units right from the common function. $f(x) = (x - 5)^2$
- $\underline{\boldsymbol{h(x) = f(x + c)}}$ This is a horizontal shift **Left** c unit. The light blue represents a shift 2 units left from the common function. $f(x) = (x + 2)^2$

Show on desmso.

Example 1A) Name the common function

- Compare the following graphs and describe what happens from the common function $f(x) = |x|$. Graph and label $g(x), h(x), and\ k(x)$.
- a) $g(x) = |x| - 4$
- b) $h(x) = |x - 3|$ right 3 units
- c) $k(x) = |x + 2| + 4$



Example 1B

- Write the equation for the function $f(x) = x^2$, resulting from a vertical shift 3 units downward and a horizontal shift 2 units to the right of the graph.

$$f(x) = (\underbrace{x - 2}_{\text{right } 2})^2 \underbrace{- 3}_{\text{down } 3}$$

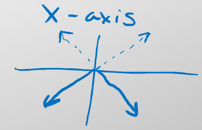
Reflecting Graphs (another rigid transformation)

Reflections can happen to a graph as well. A reflection in the graph of $y = f(x)$ has two options:

- 1. $h(x) = -f(x)$ has a reflection about the x-axis
- 2. $h(x) = f(-x)$ has a reflection about the y-axis
- Let $f(x) = |x|$. Describe the graph of $g(x) = -|x|$ in terms of f.

$g(x) = -f(x)$

[reflection over the x-axis]



- What about $f(x) = |-x|$?

$h(x) = f(-x) = |x|$

  - Show on desmso WITH SQUARE ROOTS.

Try it out!

$(-2)^3 = -2^3$ $\quad$ $(-2)^2 = 2^2$

- **Example 2.** $f(x) = x^3 + 3$. Describe the reflections in the graph of f generated by the following functions.
- **a)** $g(x) = -x^3 + 3$.

$= (-x)^3 + 3$

Reflection over y-axis

- **b)** $h(x) = -x^3 - 3$.

$= -1(\underbrace{x^3+3})$

$= -1(f(x))$

$= -f(x)$

Reflection over x-axis

Rigid vs Non-rigid Transformations

- **There are three rigid transformations:**
- They are: Shifts (H & V) and reflections. These only change the position of the graph.
- **Nonrigid transformations** actually distort the shape of the graph, instead of just shifting or reflecting it.
- Nonrigid transformations of $y = f(x)$ come from equations of the form

$y = cf(x)$. If $c > 1$, then there is a vertical **stretch** of the graph of $y = f(x)$.

If $0 < c < 1$, then there is a vertical **shrink (squish)**. (Sometimes called a horizontal stretch.)

Non-rigid Transformations

- So if c is a fraction (or decimal) less than 1 and greater than zero, then we have a vertical Shrink.
- If c is greater than 1, then we have a vertical stretch.
- Examples of a **vertical shrink**: $f(x) = \frac{1}{4}x^3$ or $f(x) = .4x^2$
- Show on desmso.

- Example of a **vertical stretch**: $h(x) = 4|x|$
- Show on desmso.

Parent Function Manipulation

- Most of the parent functions can be written in a "vertex format" style which describes all of the possible transformation.
- They all will follow the same pattern.
- a is reflections (if negative), stretch and shrink.
- k is a vertical shift.
- h is a horizontal shift (remember that this will shift the opposite of it's sign).
- $f(x) = ax + k$ — *Linear function*
- $f(x) = a|x - h| + k$ — Absolute Value function — vertex (h, k)
- $f(x) = a\sqrt{x - h} + k$ — (h,k) — Square root function
- $f(x) = a(x - h)^2 + k$ — Quadratic Function — vertex (h, k)
- $f(x) = a(x - h)^3 + k$ — Cubic function — point of inflection (h, k)
- $f(x) = \frac{a}{x-h} + k$ — Rational Function — Show on desmso.

All "Important" points start @ $(0,0)$.

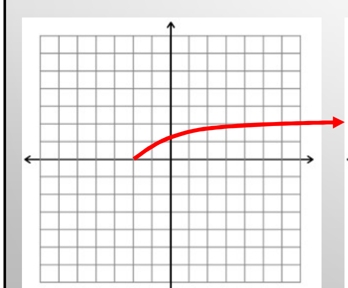
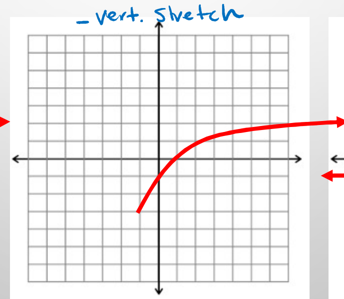
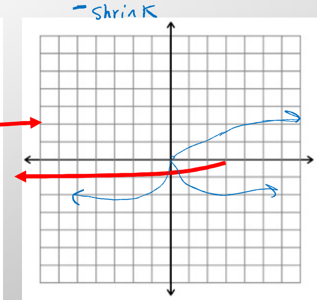
You try.

- Identify the common function.
- Describe what rigid and non-rigid transformations are occurring to the common function in each graph.

$f(x) = \sqrt{x}$
hor. shift left 2 units.

$f(x) = \sqrt{x}$
- vert down 3 unit
- hor. left 1 unit
- vert. stretch

$f(x) = \sqrt{x}$ &
- reflection x-axis y-axis
- hor. shift right 3 units.
- shrink

Example

- Find the common function, Identify the transformation(s) and roughly sketch the following functions (when sketching, graph both the common function and transformed function.)

$3 - x^3 + 3$

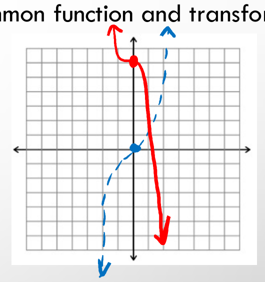
- 1) $f(x) = (3 - x^3) + 3$

$= -x^3 + 3 + 3$

$= -x^3 + 6 \longrightarrow -1(\underbrace{x^3 - 6}_{\text{shift}})$

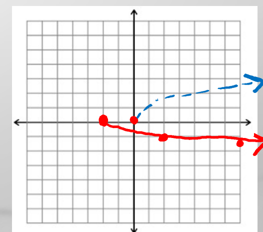
$(-x)^3 + 6$

then reflect



- 2) $h(x) = -\frac{1}{2}\sqrt{x+2}$

x	y
-2	0
2	-1
7	-3/2



Homework

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