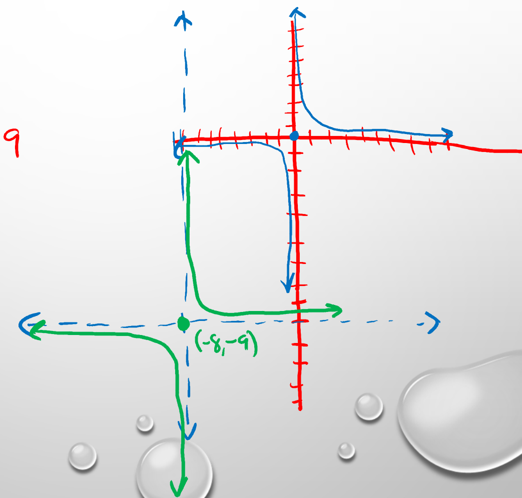
PAGE: 47 #5-17ODD, 23-43EOO, 51-61ODD, 74

59) Rational function

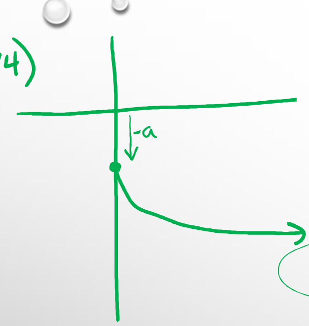
c) $\frac{1}{x} \rightarrow \frac{1}{x+8} - 9$

$f(x) = \frac{1}{x}$

d) $g(x) = f(x+8) - 9$



74)



$\sqrt{x} - a$

$-\sqrt{x} - a$

$-\sqrt{x} - 4$ (a)

$-4 - \sqrt{x}$ (b)

$\cancel{-4 - \sqrt{-x}}$ ~~(c)~~

$\cancel{\sqrt{-x} - 4}$ ~~(d)~~

$\cancel{\sqrt{-x} + 4}$ ~~(e)~~

$\cancel{\sqrt{x} - 4}$ ~~(f)~~

55 d) $x^3 = f(x)$ $\quad g(x) = \frac{1}{3}(x-2)^3$

$\left(\frac{1}{3}\right)(x-2)^3 = f(x-2)\left(\frac{1}{3}\right)$

$\frac{1}{3}(x-2)^3 = \boxed{\frac{1}{3}f(x-2)}$

$2(2x+7)^2$

$2f(g(x))^2$

1.5 COMBINATIONS OF FUNCTIONS

OBJECTIVE: IN THIS LESSON YOU LEARN HOW TO FIND ARITHMETIC COMBINATIONS AND COMPOSITIONS OF FUNCTIONS.

<u>ARITHMETIC</u> COMBINATIONS OF FUNCTIONS

- TWO FUNCTIONS CAN BE COMBINED BY THE OPERATIONS OF <u>ADDITION, SUBTRACTION, MULTIPLICATION, AND DIVISION</u> TO CREATE NEW FUNCTIONS.

- THE DOMAIN OF AN ARITHMETIC COMBINATION OF FUNCTIONS *F* AND G CONSISTS OF . . . ALL REAL NUMBERS THAT ARE COMMON TO THE DOMAINS OF *F* AND G. IN THE CASE OF THE QUOTIENT $f(x)/g(x)$, THERE IS THE FURTHER RESTRICTION THAT $g(x) \neq 0$.
- <u>SAPUTO SAYS</u>: FOLLOW THE RESTRICTIONS OF $f(x), g(x)$ AND THEIR COMBINATION.

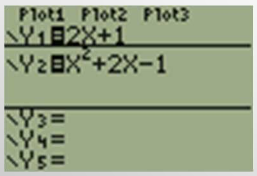
THE OPERATIONS OF FUNCTIONS

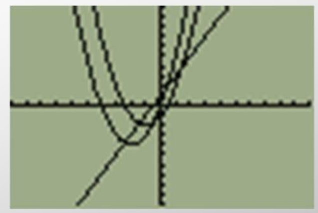
Example: $f(x) = 2x, \quad g(x) = x^2$

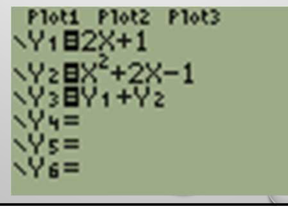
- ADDITION: $(f + g)(x)$ $f(x) + g(x)$ $2x + x^2$ D: $\mathbb{R}$
- SUBTRACTIONS: $(f - g)(x)$ $f(x) - g(x)$ $2x - x^2$ D: $\mathbb{R}$
- MULTIPLICATION: $(fg)(x)$ $f(x) \cdot g(x)$ $2x \cdot x^2 = 2x^3$ D: $\mathbb{R}$
- DIVISION: $\left(\frac{f}{g}\right)(x)$ $\frac{f(x)}{g(x)} = \frac{2x}{x^2} = \frac{2}{x}$ D: $\mathbb{R}$ $x \neq 0$

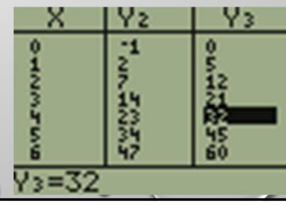
GRAPHING UTILITY USE… (IF WE HAVE TIME)

- FOLLOW ALONG TO SEE WHAT TO DO TO SOLVE FOR A FUNCTION:
- EXAMPLE $f(x) = 2x + 1$
- $g(x) = x^2 + 2x - 1$









ALGEBRAICALLY

- EXAMPLES: STATE THE DOMAIN AS WELL.

$f(x) = 2x + 1$ & $g(x) = x^2 - 1$

1) FIND $(f - g)$ $2x+1-(x^2-1) = \boxed{2x+1-x^2+1 = -x^2+2x+2}$ D: ℝ
2) FIND $(f + g)$ $= 2x+1+x^2-1 = \boxed{x^2+2x}$ D: ℝ
3) FIND (fg) $= (2x+1)(x^2-1) = \boxed{2x^3-2x+x^2-1}$ D: ℝ
4) FIND $\left(\frac{f}{g}\right)$ $= \boxed{\frac{2x+1}{x^2-1} = \frac{2x+1}{(x-1)(x+1)}}$ D: ℝ, $x \neq \pm 1$

EXAMPLE

- LET $f(x) = 7x - 5$ AND $g(x) = 3 - 2x$. FIND $(f - g)(4)$.
- METHOD #1) COMBINE FIRST

$f(x) - g(x) = 7x - 5 - (3 - 2x)$
$7x - 5 - 3 + 2x$
$9x - 8 \rightarrow D: \mathbb{R}$
$(f - g)(4) = 9(4) - 8$
$= 36 - 8$
$\boxed{= 28}$

METHOD #2) SOLVE FIRST

$7(4) - 5 - (3 - 2(4))$
$28 - 5 - (3 - 8)$
$23 - (-5)$ D: ?
$\boxed{28}$

YOU DO THIS ON YOUR OWN: YDTOYO

D: $\mathbb{R}$ $x \neq 0$ D: $x \geq 1$

- $f(x) = \frac{1}{x}$, $g(x) = \sqrt{x - 1}$
- FIND THE FUNCTION AND THE DOMAIN

a) $(fg)(x) = \boxed{\dfrac{\sqrt{x-1}}{x}}$ D: $x \geq 1$ ($x = 0$ is redundant)

R) $\left(\frac{f}{g}\right)(x) = \dfrac{\frac{1}{x}}{\sqrt{x-1}} = \boxed{\dfrac{1}{x\sqrt{x-1}}}$ D: $x > 1$

$\dfrac{\frac{1}{a}}{b} = \dfrac{1}{ab}$

COMPOSITION OF FUNCTIONS

- THE COMPOSITION OF THE FUNCTION F WITH THE FUNCTION G IS
- $(f \circ g)(x) = f(g(x))$

- THE DOMAIN OF $(f \circ g)$IS THE SET OF ALL X IN THE DOMAIN OF G SUCH THAT $g(x)$ IS IN THE DOMAIN OF F.

- *WE USE* $g(x)$ AS THE X VALUE IN $f(x)$.

- FOR TWO FUNCTIONS *F* AND G, TO FIND $(f \ o \ g)(x)$ ... REPLACE EACH OCCURRENCE OF *X* IN *F* WITH THE ALGEBRAIC EXPRESSION WHICH DEFINES G(X). THEN SIMPLIFY.

- EXAMPLE:

- LET $f(x) = 3x + 4$ AND LET $g(x) = 2x^2 - 1$. FIND $(\boldsymbol{a})\ (f \ o \ g)(x)$ and $(\boldsymbol{b})\ (g \ o \ f)(x)$ and THEIR DOMAINS.

a) $3(2x^2-1)+4$

$6x^2-3+4$

$6x^2+1$

$D: \mathbb{R}$

b) $2(3x+4)^2 - 1$

$2(9x^2+24x+16) - 1$

$18x^2+48x+32-1$

$18x^2+48x+31$

$D: \mathbb{R}$

EXAMPLES:

- USE $f(x) = x^2 + 2x$ AND $g(x) = \sqrt{x+1}$, FIND THE EQUATION AND DOMAIN OF:
- $(g \circ f) = g(f(x)) = \sqrt{(x^2+2x)+1} = \sqrt{x^2+2x+1}$ D: $\mathbb{R}$

  $= \sqrt{(x+1)^2} = \boxed{x+1}$
- $(f \circ g) = \left(\sqrt{x+1}\right)^2 + 2\sqrt{x+1}$

  $\boxed{= x+1 + 2\sqrt{x+1}}$ D: $\mathbb{R}$, $x \geq -1$

YOU TRY SOME MORE:

- $f(x) = \frac{3x}{x+4}$ & $g(x) = 2x - 4$
- FIND: $(f \circ g)$ $\frac{3(2x-4)}{(2x-4)+4} = \frac{6x-12}{2x} = \frac{3x-6}{x}$ D: $\mathbb{R}$ $x \neq 0$

  or $= 3 - \frac{6}{x}$

EXAMPLE

- 1) $h(x) = (x + 1)^2 - x - 1$. FIND TWO FUNCTIONS f AND g, SUCH THAT $h(x) = (f \circ g)(x)$.

$f(g(x))$

$(g(x))^2 - (g(x))$

$g(x) = x + 1$

$f(x) = x^2 - x$

- 2) YOU TRY: $h(x) = \frac{x+3}{(x+3)^2-4}$, FIND f and g, SUCH that $h(x) = f(g(x))$

$g(x) = x + 3$

$f(x) = \frac{x}{x^2 - 4}$

$D: (x+3)^2 - 4 \neq 0$

$\sqrt{(x+3)^2} = \sqrt{4}$

$x + 3 = \pm 2$

$x = 2 - 3$ & $x = -2 - 3$

- CHALLENGE: FIND THE DOMAIN

$\mathbb{R}, x \neq -1, -5$

EXAMPLE: USES IN BUSINESS

A DEMAND FUNCTION FOR A CERTAIN PRODUCT IS

- $p = f(x) = 50 - 0.05x$

WHERE *P* IS THE PRICE OF THE PRODUCT WHEN *X* UNITS ARE SOLD.

THE COST OF PRODUCING *X* UNITS IS GIVEN BY

- $C(x) = 0.09x + 12{,}000.$

a) FIND THE REVENUE FUNCTION, $R(x) = xf(x)$.

$R(x) = 50x - 0.05x^2$

B) FIND THE **PROFIT** FUNCTION $P(x) = R(x) - C(x)$

$P(x) = (50x - 0.05x^2) - (.09x + 12{,}000) = \boxed{-0.05x^2 + 49.91x - 12{,}000}$

BACTERIA COUNT: FROM BOOK P.55

- THE NUMBER N OF BACTERIA IN A REFRIGERATED PETRI DISH IS GIVEN BY

$$N(T) = 20T^2 - 80T + 500, \qquad 2 \leq T \leq 14$$

WHERE T IS THE TEMPERATURE OF THE PETRI DISH (IN $C°$). WHEN THE PETRI DISH IS REMOVED FROM REFRIGERATION, THE TEMPERATURE OF THE DISH IS GIVEN BY $T(t) = 4t + 2, \quad 0 \leq t \leq 3$, WHERE t IS THE TIME (HOURS).

a) FIND THE COMPOSITION $N(T(t))$ AND INTERPRET IT'S MEANING IN CONTEXT

b) FIND THE NUMBER OF BACTERIA IN THE PETRI DISH AFTER 2 HOURS.

c) FIND THE TIME WHEN THE BACTERIA COUNT REACHES 2000.

a) $N(T(t)) = 20(4t+2)^2 - 80(4t+2) + 500$

$= 20(16t^2 + 16t + 4) - 320t - 160 + 500$

$= 320t^2 + 320t + 80 - 320t - 340$

$\boxed{N(T(t)) = 320t^2 + 420}$

Bacteria growth as a function of time.

b) $320(2)^2 + 420$

$1280 + 420$

$N(T(2)) = 1700$

c) $2000 = 320t^2 + 420$

$-420 \qquad\qquad -420$

$\frac{1580}{320} = \frac{320t^2}{320}$

$\pm\sqrt{\frac{79}{16}} = \sqrt{t^2}$ (neg won't work

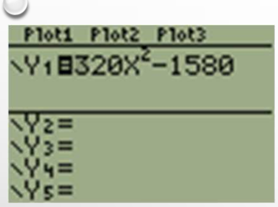
graphing method

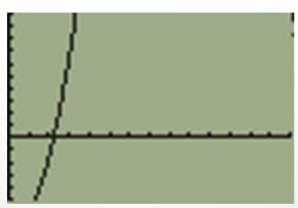
$\frac{\sqrt{79}}{4} \approx 2.22$ hours.

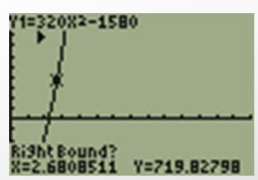
GRAPHING METHOD TO SOLVE FOR t

$$0 = 320t^2 - 1580$$

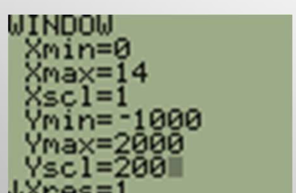
Question at hand: find when $y = 0$ for t between 2 and 14.

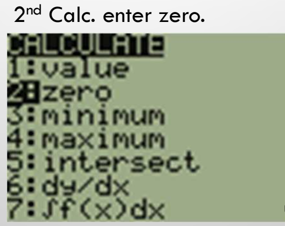


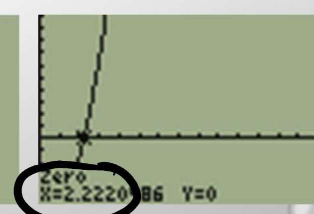




2nd Calc. enter zero.







$t \approx 2.22$ hours

$Do\ f\left(g(x)\right)$,
WHERE $g(x) = homework, f(x) = you$

- PS1.5 COMBOS P.56 #13-31ODD 41-44, 45, 47, 53,67, 71-77ODD, 84, 85