Warm-up:

Solve each quadratic equation by factoring.

1) $4x^2 - 4x - 3 = 0$

12·1
6·2
4·3

$(4x^2 - 6x) + (2x - 3) = 0$

$2x \qquad 1$

$2x(2x-3) + 1(2x-3) = 0$

$(2x-3)(2x+1) = 0$

$2x - 3 = 0 \quad 2x + 1 = 0$

$x = 3/2$

$x = -1/2$

2) $x^2 + 3x - 18 = 0$

18·1
9·2
6·3

$(x + 6)(x - 3) = 0$

$x + 6 = 0 \quad x - 3 = 0$

$x = -6$ & $x = 3$

3) $x^2 - 64 = 0$

DO $(x)^2 - (8)^2 = 0$

$(x-8)(x+8) = 0$

$x - 8 = 0 \quad x + 8 = 0$

$x = 8$ & $x = -8$

4) $2x^2 = -8x$

DO set = to 0

$2x^2 + 8x = 0$

$2x(x+4) = 0$

$2x = 0 \quad x + 4 = 0$

$x = 0$ & $x = -4$

Agenda:

1) **Warm-up**
2) **Homework questions**
3) **New Lesson---3.2 Complex Numbers**
4) **Homework/in-class activity**

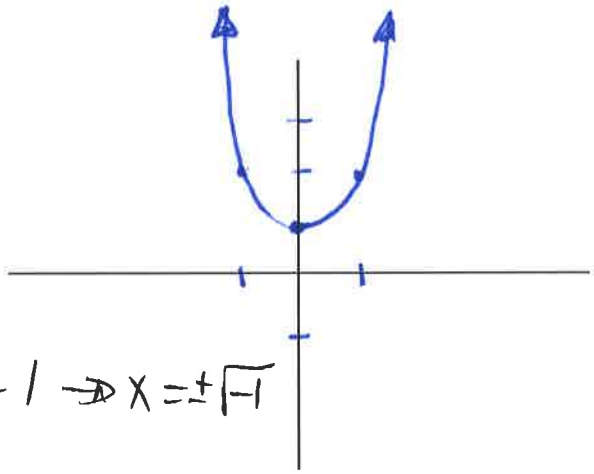
Algebra 2: 3.2 Complex Numbers:

Learning Targets for Today:

1) Define and use the imaginary i.
2) Add, Subtract, and Multiply complex numbers.
3) Find complex solutions and zeros.

Not all Quadratic equations have real-number solutions!
For instance $x^2 + 1 = 0$

***lets look at the graph!



NO x-intercepts = NO real solutions

$x^2+1=0 \rightarrow x^2=-1 \rightarrow x=\pm\sqrt{-1}$

*notice: $x^2 + 1$ has no real number solutions (even by the square-root method b/c $\sqrt{-1}$ has no solutions....
To overcome this problem, mathematicians created an expanded system of numbers using the imaginary number i.

It is not real because it is not possible to take $\sqrt{-}$
So...

(A) Imaginary Unit i

$i = \sqrt{-1} = i$

$i^2 = \left(\sqrt{-1}\right)^2 = \sqrt{-1} \cdot \sqrt{-1} = -1$

$i^2 = \left(\sqrt{-1}\right)^2 = \sqrt{-1} \cdot \sqrt{-1} = -1$

OR $\sqrt{16} = \sqrt{4 \cdot 4} = \sqrt{4} \cdot \sqrt{4} = 2 \cdot 2 = 4$

$\sqrt{20} = \sqrt{4 \cdot 5} = \sqrt{4} \cdot \sqrt{5} = 2\sqrt{5}$

Example: (show them how to break up a square-root: $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$

1) $\sqrt{-25}$

OR $\sqrt{-1 \cdot 25}$

$\sqrt{25} \cdot \sqrt{-1}$

$5i$

2) $\sqrt{-12}$

$\sqrt{12 \cdot -1}$

$\sqrt{12} \cdot \sqrt{-1}$

$\sqrt{12} \cdot i$

$\sqrt{12}\, i$

$\sqrt{4} \cdot \sqrt{3}\, i$

$2\sqrt{3}\, i$

Try: simplify

1) $\sqrt{-16}$

$\sqrt{16 \cdot -1} \Rightarrow \sqrt{16} \cdot \sqrt{-1}$

$4i$

3) $\sqrt{-18}$

$\sqrt{18 \cdot -1} \Rightarrow \sqrt{18} \cdot \sqrt{-1} \rightarrow \sqrt{18}\, i$

$\sqrt{18} \cdot i$

$\sqrt{9 \cdot 2}\, i \Rightarrow \sqrt{9} \cdot \sqrt{2}\, i \rightarrow 3\sqrt{2}\, i$

(B) Complex Numbers

standard form a + bi (a: number Part; b: imaginary Part)

Example:

1) 2 + 3i

2) 3i or $0 + 3i$

3) 4

or

$4 + 0i$

5) 2-i

(C) Adding and Subtracting Complex Numbers

***just add the real number parts together...then add the imaginary parts**

Example:

1) $(4 - i) + (3 + 2i) =$ $4+3 = 7$, $-i+2i = i$ $\rightarrow 7+i$

Example:

2) $(7 - 5i) - (1 - 5i)$ distribute 1st

$7 - 5i - 1 + 5i$

$6 + 0i \rightarrow 6$

3) $6 - (-2 + 9i) + (-8 + 4i)$

$6 + 2 - 9i - 8 + 4i$

$8 - 9i - 8 + 4i \rightarrow 0 - 5i$ or $-5i$

Try:

1) $(-1 + 2i) + (3 + 3i)$

$2 + 5i$

2) $(2 - 3i) - (3 - 7i)$

$2 - 3i - 3 + 7i$

$-1 + 4i$

3) $2i - (3 + i) + (2 - 3i)$

$2i - 3 - i + 2 - 3i$

$-2i - 1 \rightarrow -1 - 2i$

(D) Multiplying Complex Numbers

Ex:

1) $5i(-2 + i)$ — just like $3x(x + 2)$ distribute

$-10i + 5i^2$ — $3x^2 + 6x$

$i^2 = -1$ [You can't leave i^2 in your answer]

$-10i + 5(-1)$ → $\boxed{-5 - 10i}$

Examples:

1) $(7 - 4i)(-1 + 2i)$ — just like $(x + 2)(x - 3)$ FOIL!!!

$-7 + 14i + 4i - 8i^2$ — $x^2 - 3x + 2x - 6$

$-7 + 18i - 8(-1)$ — $x^2 - x - 6$

$-7 + 18i + 8$

$1 + 18i$

2) $(6 + 3i)(6 - 3i)$ — special type....conjugates

$36 - 18i + 18i - 9i^2$

36 this is why we like $-9(-1)$

$36 + 9$ → $\boxed{45}$

Try:

1) $-i(3+i)$

$-3i-i^2$

$-3i-(-1)$

$1-3i$

2) $(2+3i)(-6-2i)$

$-12-4i-18i-6i^2$

$-12-22i-6(-1)$

$-12-22i+6 \rightarrow -6-22i$

3) $(1+2i)(1-2i)$

$1-2i+2i-4i^2$

$1-4(-1)$

$1+4 \rightarrow 5$

(E) Solving Quadratic Equations

Step 1: isolate the "squared" part

Step 2: take the square-root of both sides...watch for imaginary answer

Example:

1) $3x^2+10=-26$

$-10 \quad -10$

$\frac{3x^2}{3}=\frac{-36}{3}$

$\sqrt{x^2}=\sqrt{-12}$

$x=\pm\sqrt{-12} \rightarrow \pm\sqrt{12\cdot -1} \rightarrow \pm\sqrt{12}\,i$

$\sqrt{4\cdot 3}\,i \rightarrow \pm\sqrt{4}\sqrt{3}\,i \rightarrow \pm 2\sqrt{3}\,i$

Try:

1) $x^2 = -4$

$\sqrt{x^2} = \sqrt{-4}$

$x = \pm\sqrt{-4}$

$x = \pm\sqrt{4}\,i$

$\boxed{x = \pm 2i}$

2) $x^2 = -18$

$\sqrt{x^2} = \sqrt{-18}$

$x = \pm\sqrt{18}\,i$

$\sqrt{9}\sqrt{2}\,i$

$\boxed{x = \pm 3\sqrt{2}\,i}$

3) $2x^2 + 9 = -41$

$-9 \quad -9$

$\frac{2x^2}{2} = \frac{-50}{2}$

$x^2 = -25$

$\sqrt{x^2} = \sqrt{-25}$

$x = \pm\sqrt{-25}$

$x = \pm\sqrt{25}\,i$

$\boxed{x = \pm 5i}$

4) $9(x-4)^2 + 1 = 0$

$-1 \quad -1$

$\frac{9(x-4)^2}{9} = \frac{-1}{9}$

$(x-4)^2 = \frac{-1}{9}$

$\sqrt{(x-4)^2} = \sqrt{\frac{-1}{9}}$

$x - 4 = \pm\sqrt{\frac{-1}{9}}$

$x - 4 = \pm\sqrt{\frac{1}{9}} \cdot \sqrt{-1}$

$\pm\frac{1}{\sqrt{9}} \cdot i$

$x - 4 = \pm\frac{1}{3}i \rightarrow \boxed{4 \pm \frac{1}{3}i}$

Homework: pg 108 (5-9, 21-27, 37-41, 49-53)