

Warm-up:

Solve each quadratic equation by factoring.

1) $x^2 + 3x + 2 = 0$

$$(x + 2)(x + 1) = 0$$

$$x + 2 = 0 \quad x + 1 = 0$$

$$x = -2 \quad \& \quad x = -1$$

2) $2x^2 - 11x = -12$

$$+12 \quad +12$$

$$2x^2 - 11x + 12 = 0$$

$$(2x^2 - 8x) \left(\frac{-3x + 12}{-3} \right) = 0$$

$$2x(x - 4) - 3(x - 4) = 0$$

$$(x - 4)(2x - 3) = 0$$

$$\begin{array}{l} 24 \cdot 1 \\ 12 \cdot 2 \\ \underline{8 \cdot 3} \\ 6 \cdot 4 \end{array}$$

$$\begin{array}{l} x - 4 = 0 \\ 2x - 3 = 0 \\ 2x = 3 \\ x = 4 \\ x = \frac{3}{2} \end{array}$$

Multiply the Complex Number Together:

3) $(7 - 4i)(-1 + 2i)$ *Foil!*

$$-7 + 14i + 4i - 8i^2$$

$$-7 + 18i - 8i^2 \quad i^2 = -1$$

$$-7 + 18i - 8(-1)$$

$$-7 + 18i + 8$$

$$1 + 18i$$

Solve the quadratic equation by the square root method:

4) $2x^2 + 9 = -41$

$$\frac{2x^2}{2} = \frac{-50}{2}$$

$$x^2 = -25$$

$$\sqrt{x^2} = \sqrt{\pm -25}$$

$$x = \sqrt{25} \cdot \sqrt{-1}$$

$$x = \pm 5i$$

Agenda:

- 1) Warm-up
- 2) Homework questions
- 3) New Lesson---3.3 Completing the Square
- 4) Homework/in-class activity

Algebra 2: 3.3 Completing the Square:

Learning Targets for Today:

- 1) Solve quadratic equations using square roots.
- 2) Solve quadratic equations by completing the square.
- 3) Write quadratic equations in vertex form.

(A) Solving a Quadratic Equation using Square Roots

We have solved quadratics by using the square-root method before:

Step 1: Isolate the squared part

Step 2: Take the square-root of both sides

Try:

$$1) \quad 3(x-1)^2 - 2 = 73$$

$$\begin{array}{r} 3(x-1)^2 = 75 \\ \hline 3 \qquad \qquad 3 \\ \hline (x-1)^2 = 25 \end{array}$$

$$x = 6 \text{ ; } -4$$

$$\begin{array}{r} x-1 = 5 \text{ ; } -5 \\ \hline +1 \quad +1 \quad +1 \end{array}$$

Our main goal today is to take quadratic equations from standard form to vertex form:

$$x^2 + 2x + 3 \longrightarrow (x + 1)^2 + 2$$

Because vertex form is way easier to graph from and you can solve by the square-root method!

- This process is called **Completing the Square!**

(B) Completing the Square:

Completing the Square:

Step 1: The leading coefficient must be 1. (if it is not, then divide by the number in front of the x^2)

Step 2: Write your equation in the correct form:

$$a(x^2 + bx + c)$$

Step 3: Take half of b ...then "square" it....add & subtract it

Step 4: Move the negative one outside of the ()

Step 5: Factor the ()

Example:

$$x^2 + 10x - 3$$

Step 1: a must be 1
(it is!)

Step 2: Correct form

Step 3: half of "b"

Step 4: move negative one outside

Step 5: Factor

$$x^2 + 10x - 3$$

$$(x^2 + 10x + 25 - 25) - 3 - 25$$

$$(5)^2 = 25$$

$$(x+5)^2 - 28$$

Vertex: $(-5, -28)$

Try/Together:

$$x^2 + 6x - 8$$

step 1
step 2

$$(x^2 + 6x + 9 - 9) - 8$$

$$(x^2 + 6x + 9) - 8 - 9$$

$$(x+3)^2 - 17$$

Try:

$$1) x^2 + 2x$$

$$(x^2 + 2x + 1 - 1)$$

$$(x+1)^2 - 1$$

$$2) x^2 - 8x + 11$$

$$(x^2 - 8x + 16 - 16) + 11$$

$$(x^2 - 8x + 16) - 5$$

$$(x-4)^2 - 5$$

$$3) x^2 + 7$$

↑
It's already in vertex form. No regular x

a is not 1

Example:

↓
 $3x^2 - 6x + 8 = 0$

step 1
&
step 2

$$\begin{aligned} & (3x^2 - 6x) + 8 = 0 \\ 3(x^2 - 2x + 1 - 1) - 3 + 8 &= 0 \\ & \begin{array}{c} \uparrow \\ (1)^2 = 1 \end{array} \quad \begin{array}{c} \downarrow \\ \text{be careful!} \end{array} \\ 3(x^2 - 2x + 1) + 5 &= 0 \\ \underline{\underline{3(x-1)^2 + 5 = 0}} \end{aligned}$$

Try/Together:

$$5x^2 - 10x + 16 = 0$$

$$\begin{aligned} & (5x^2 - 10x) + 16 = 0 \\ 5(x^2 - 2x + 1 - 1) - 5 + 16 &= 0 \\ & \begin{array}{c} \uparrow \\ (1)^2 = 1 \end{array} \\ 5(x-1)^2 + 11 &= 0 \\ \underline{\underline{5(x-1)^2 + 11 = 0}} \end{aligned}$$

Try:

$$1) 3x^2 - 12x + 16 = 0$$

$$\left(\frac{3x^2 - 12x}{3} \right) + 16 = 0$$

$$3(x^2 - 4x + 4 - 4) - 12 + 16 = 0$$

$(2)^2 = 4$

$$3(x^2 - 4x + 4) + 4 = 0$$

$$\underline{\underline{3(x-2)^2 + 4 = 0}}$$

$$2) 3x^2 - 12x + 1$$

$$(3x^2 - 12x) + 1$$

$$3(x^2 - 4x + 4 - 4) + 1$$

$(2)^2 = 4$

$$3(x^2 - 4x + 4) - 11$$

$$\underline{\underline{3(x-2)^2 - 11}}$$

*Right now we can complete the square...so we can find the vertex and slope...but it also helps us solve a quadratic (find x-int)

Solving by Completing the Square:

Step 1: Complete the Square:

- this should put your equation into this form: $a(x-h)^2+k$

Step 2: isolate the squared part ()²

Step 3: take the square-root of both sides

- should get two answers

Example:

$$x^2 - 10x - 3 = 0$$

$$(x^2 - 10x + 25 - 25) - 3 = 0$$

(5)² = 25

$$(x^2 - 10x + 25) - 28 = 0$$

$$(x-5)^2 - 28 = 0$$

$$\sqrt{(x-5)^2} = \sqrt{28}$$

$$x-5 = \pm \sqrt{28}$$

$$x-5 = \pm 2\sqrt{7}$$

+5 +5

$$x = 5 \pm 2\sqrt{7}$$

*Now I
Solve!*

$$\begin{array}{l} \sqrt{28} \\ \sqrt{7 \cdot 4} \\ \sqrt{4} \cdot \sqrt{7} \\ 2\sqrt{7} \end{array}$$

Try:

$$x^2 + 2x - 9 = 0$$

$$(x^2 + 2x + 1 - 1) - 9 = 0$$

$(1)^2 = 1$

$$(x^2 + 2x + 1) - 10 = 0$$

Now solve

$$(x+1)^2 - 10 = 0$$

$$\sqrt{(x+1)^2} = \sqrt{10}$$

$$x+1 = \pm \sqrt{10}$$

$$x = -1 \pm \sqrt{10}$$

$$\rightarrow \boxed{x = -1 \pm \sqrt{10}}$$

Example:

$$2x^2 - 12x + 14 = 0$$

$$(2x^2 - 12x) + 14 = 0$$

$$2(x^2 - 6x + 9 - 9) - 18 + 14 = 0$$

$(3)^2 = 9$

$$2(x^2 - 6x + 9) - 4 = 0$$

$$2(x-3)^2 - 4 = 0$$

$$2(x-3)^2 = 4$$

$$(x-3)^2 = 2$$

$$x-3 = \pm \sqrt{2}$$

$$\boxed{x = 3 \pm \sqrt{2}}$$

"OR"

$$2x^2 - 12x + 14 = 0 \quad 7.1$$

$$2(x^2 - 6x + 7) = 0$$

$$2\left(x - \frac{\text{nope}}{x} - \right) = 0$$

Try:

$$-3x^2 - 18x - 6 = 0$$

$$(-3x^2 - 18x) - 6 = 0$$

$$-3(x^2 + 6x + 9 - 9) - 27 - 6 = 0$$

$(3)^2 = 9$

$$\rightarrow -3(x^2 + 6x + 9) + 21 = 0$$

$$-3(x+3)^2 + 21 = 0$$

$$-3(x+3)^2 = -21$$

$$(x+3)^2 = 7$$

$$x+3 = \pm \sqrt{7}$$

$$\boxed{x = -3 \pm \sqrt{7}}$$