

Math Encounters – Chapter 7 Review

Show the set-up (fraction, combination, etc.) AND final answer to get full credit. Reduce final fractions to lowest terms unless otherwise noted.

The statistics from the last 105 NFL games played by Joe Quarterback are shown below. The status of his team at half-time (note a shut-out means they were behind with no points at half-time) was compared to the overall result for his team at the end of the game (they lost or won). If a game is chosen at random, find the probabilities below. Give exact fractions and percentages rounded to the nearest whole.

Half-Time	Loss	Win	Total
Leading	14	18	32
Tied	11	13	24
Behind	12	22	34
Shut-out	8	7	15
Total	45	60	105

1. P(win given they were behind at the half) $\frac{22}{34} = \frac{11}{17}$

2. P(win) $\frac{60}{105} = \frac{12}{21} = \frac{4}{7}$

3. P(leading at the half) $\frac{32}{105}$

4. P(leading or tied at the half) $\frac{32 + 24}{105} = \frac{56}{105} = \frac{8 \cdot 7}{15 \cdot 7} = \frac{8}{15}$

5. P(win given they were behind at the half)
Same as #1

6. P(behind at the half given they won)
 $\frac{22}{60} = \frac{11}{30}$

7. P(were leading at the half and won)
 $\frac{18}{32} = \frac{9}{16}$

8. P(were tied and leading at the half)
0 (CANT HAPPEN)

9. A pair of standard dice is tossed
a. How many outcomes are possible? $6 \cdot 6 = 36$

b. List all of the outcomes that have a SUM of 7 (1,6) (2,5) (3,4) (4,3) (5,2) (6,1)

c. What is the probability that the SUM of the two numbers shown will be 7? $\frac{6}{36} = \frac{1}{6}$

d. What is the probability that the sum of the two numbers will NOT be 7?
 $36 - 6 = 30 = \frac{30}{36} = \frac{5}{6}$ or $1 - \frac{1}{6} = \frac{5}{6}$

In a standard 52 card deck, what is the probability of drawing a:

10. Seven $\frac{4}{52} = \frac{1}{13}$

11. Black Heart
0

12. NOT a King of Diamonds
 $\frac{51}{52}$

13. Red Face Card
 $\frac{6}{52} = \frac{3}{26}$

14. Jack and Spade
 $\frac{1}{52}$

15. Jack or Spade
 $\frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}$

16. Six or Seven
 $\frac{4+4}{52} = \frac{8}{52} = \frac{2}{13}$

17. Ten given you have drawn a heart
 $\frac{1}{13}$

18. Heart given you have drawn a red card
 $\frac{13}{26} = \frac{1}{2}$

19. Queen give you have drawn a face card
 $\frac{4}{12} = \frac{1}{3}$

20. Two marbles are drawn from a small box that contains ¹⁴fourteen marbles. Four of the marbles are green, five are red, two are yellow, and three are blue. Find the probability the two marbles drawn are both red if:
- a. the two marbles are selected from the box with replacement
- b. the two marbles are selected from the box without replacement

$$\left(\frac{5}{14}\right)\left(\frac{5}{14}\right) = \frac{25}{196}$$

$$\left(\frac{5}{14}\right)\left(\frac{4}{13}\right) = \left(\frac{5}{7}\right)\left(\frac{2}{13}\right) = \frac{10}{91}$$

21. A bag of marbles contains ten marbles: one white, two blue, three red, and four yellow. Find the probabilities or odds described if each question starts with all ten marbles in the bag.

Parts a – e are to be done assuming you are picking one marble with replacement

- a. P(red)

$$\frac{3}{10}$$

- b. P(blue or yellow)

$$\frac{2+4}{10} = \frac{6}{10} = \frac{3}{5}$$

- c. P(blue and yellow)

0 CANT BE BLUE AND YELLOW AT SAME TIME.

- d. the odds on favor of picking a blue marble

$$2:8$$

$$1:4$$

- e. the odds against picking a white marble

$$9:1$$

The next two conditional probabilities assume you are drawing without replacement

- f. P(white given you already drew out a white)

$$\left(\frac{1}{10}\right)\left(\frac{0}{10}\right) = 0$$

- g. P(white given you already drew out a blue)

$$\left(\frac{2}{10}\right)\left(\frac{1}{9}\right) = \frac{2}{90} = \frac{1}{45}$$

22. The odds in favor of Dorothy getting promoted are 8:5.

- a. Find the odds against Dorothy getting promoted

$$5:8$$

- b. Find the probability Dorothy is promoted

$$5+8=13 \quad \frac{8}{13}$$

- c. Find the probability Dorothy is not promoted

$$1 - \frac{8}{13} = \frac{5}{13}$$

23. A license plate is to consist of two letters followed by three digits. How many different plates can be made if repetition of letters is allowed, but we may not use the letters O, I, or L, nor the digits 0 or 1?

$$23 \cdot 23 \cdot 8 \cdot 8 \cdot 8 = 270,848$$

$$26-3=23$$

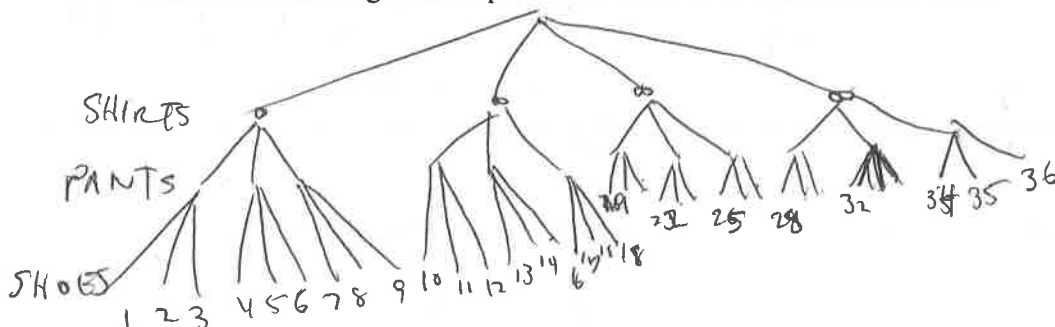
$$10-2=8$$

24. Delia has 4 shirts, 3 pants, and 3 pairs of shoes to wear to work.

- a. Use the counting principle to find the number of outfits she can create

$$4 \cdot 3 \cdot 3 = 36$$

- b. Construct a tree diagram to represent the number of outfits she can create



Find the numerical value of each of the following (showing steps used to simplify the formulas by hand)

25. $7!$

$$7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5,040$$

26. ${}_{10}C_6 =$

$$\frac{10!}{(10-6)!6!} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 210$$

27. ${}_6P_4 =$

$$\frac{6!}{(6-4)!} = \frac{6!}{2!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1} = 360$$

Show the set-up and answer to each of the following. Be sure to indicate if the problem is a combination, permutation, or other.

28. How many different ways can the letters E, L, V, I, or S be arranged?

$${}_5P_5 = \frac{5!}{(5-5)!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{0!} = \frac{120}{1} = 120$$

29. How many different ways can we choose three flavors for a sundae out of 31 flavors?

$${}_{31}C_3 = \frac{31!}{(31-3)!3!} = \frac{31 \cdot 30 \cdot 29 \cdot 28 \cdot 27 \cdot 26 \cdot 25 \cdot 24 \cdot 23 \cdot 22 \cdot 21 \cdot 20 \cdot 19 \cdot 18 \cdot 17 \cdot 16 \cdot 15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{28 \cdot 27 \cdot 26} = 4,495$$

30. How many different ways can we choose three students to be ASB Pres, VP, or Sec out of ten running?

$${}_{10}P_3 = \frac{10!}{(10-3)!} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{7!} = 720$$

31. How many different ways can we choose four books to read from a list of the top 10 sellers?

$${}_{10}C_4 = \frac{10!}{(10-4)!4!} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 210$$

32. How many different ways can we choose five students for the local spelling bee out of 30 in a class?

$${}_{30}C_5 = \frac{30!}{(30-5)!5!} = \frac{30 \cdot 29 \cdot 28 \cdot 27 \cdot 26 \cdot 25 \cdot 24 \cdot 23 \cdot 22 \cdot 21 \cdot 20 \cdot 19 \cdot 18 \cdot 17 \cdot 16 \cdot 15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{25 \cdot 24 \cdot 23 \cdot 22 \cdot 21 \cdot 20 \cdot 19 \cdot 18 \cdot 17 \cdot 16 \cdot 15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 142,506$$

33. How many different ways can we choose to give away a grand prize and a runner up to 5 applicants?

$${}_5P_2 = \frac{5!}{(5-2)!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3!} = 20$$

34. A small business has 5 men and 8 women as employees. A committee is to be formed from these employees.

a. How many committees of four people can be formed?

$${}_{13}C_4 = \frac{13!}{(13-4)!4!} = \frac{13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = \frac{17,160}{24} = 715$$

b. How many committees could be formed if it must consist of two men and two women?

$${}_5C_2 \cdot {}_8C_2 = \frac{5!}{(5-2)!2!} \cdot \frac{8!}{(8-2)!2!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3! \cdot 2} \cdot \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{6! \cdot 2} = 10 \cdot 28 = 280$$

c. What is the probability the committee will have two men and two women?

$$P(2m+2w) = \frac{280}{715} = \frac{56}{143} \approx 39.2\%$$

